



ORIGINAL RESEARCH ARTICLE

A NEW FORMULA FOR GROUNDWATER RECHARGE ESTIMATION IN YOLA, NIGERIA

A. B. Umaru^{1*}, S. Lukman², I. A. Oke³, and B. G. Umara⁴

¹Department of Agricultural and Environmental Engineering, Modibbo Adama University of Technology, Yola
Nigeria

² Civil Engineering Department, University of Hafr Al-Batin, Hafr Al-Batin, Saudi Arabia

³Department of Civil Engineering, Obafemi Awolowo University, Ile-Ife, Nigeria

⁴Department of Agricultural and Environmental Resources Engineering, University of Maiduguri, Nigeria)

* Corresponding author's email address: ababayi234@yahoo.com

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ABSTRACT

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In this paper, relationship between rainfall and groundwater recharge estimation was evaluated. A standard numerical formula that relates rainfall to groundwater recharge estimation was proposed with the aim of providing error-free groundwater estimate from rainfall. Rainfalls measured in a selected location in Yola, Nigeria and standard formula were used to fix the constants (A, B and c) in the new numerical formula using Microsoft Excel Solver. The constants were used to establish new numerical formula. The new numerical formula was used to estimate groundwater recharge from the rainfall. The accuracy of the new numerical formula was evaluated statistically using Analysis of Variance (ANOVA), relative error; the degree of accuracy, numerical reliability, Model of Selection Criterion (MSC) and Akaike Information Criterion (AIC) and compared with the previous formulae in use for field groundwater recharge estimation with Uttar Pradesh (UP) as the reference value of groundwater recharge. The study revealed that groundwater recharge estimated using the formula $(R_{gr} = 0.621(P - 1.019)^{0.814})$ was similar to groundwater recharge estimated using standard formulae (UP, Modified Chaturvedi, Kumar and Seethapathi and Rao). In all cases there were significant differences between the groundwater recharge estimated using all the formulae. The new formula provided the lowest relative error of 0.887%, the highest MSC of 4.911; the degree of accuracy of 99.113 % and the lowest AIC of 436.306. The accuracy of the formulae was in the order of new formula greater than Chaturvedi formula greater than Kumar and Seethapathi formula greater than Rao formula. It was concluded that modelling of groundwater recharge using the numerical formula is a promising tool for estimating groundwater recharge with minimum error in water resources management.

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1.0 Introduction

Moisture of soil and soil – water are important ingredients in fertility of the soil and food security. Groundwater recharge includes the drive of water from the unsaturated zone into the saturated region of the soil below the water table. The water runs together with the associated

flow pattern away from the water table within the saturated region (Freeze and Cherry, 1979; Yeh et al., 2008). Groundwater recharge occurs at a point when water containing dissolved minerals flows beyond the groundwater level and infiltrates into the saturated zone. Groundwater recharge is a tremendously important water component of the hydrological cycle in nature. Modifications in groundwater storage comprise various recharge and discharge progressions. Most important recharge sources are rainfall, recharge from rivers, recharge from ponds, recharge from irrigation fields. The crucial discharge processes are evapotranspiration, pumping, baseflow to rivers. There are several factors that can affect the existence and discrepancy of groundwater level in a region. The major and significant factors include topography, lithology, geological structures, depth of weathering, the extent of fractures, the primary and secondary porosity of the soil, slope and drainage patterns of the region, landform and land cover, environmental factors and climate (Mukherjee 1996; Jaiswal et al., 2003).

It is well known and established that ground water recharge estimation methods are numerous. It has been documented that none of these methods will give the same groundwater recharge rates when used for the same area. The use of these groundwater recharge estimate methods are limited to particular environments (humid or arid), and the availability of data for evaluation (Adams et al., 2004). The groundwater recharge methods and models in the determination of the groundwater recharge are characterized into three groups (physical model, calculated from the base flow, chemical model, measurement of water-soluble substances and numerical model, using numerical method). The numerical approaches include the computer programs such as HELP, RORA, PULSE, PART, HY SEP and Wells Water-table fluctuations (Risser et al., 2008; Seyed et al., 2013). Chandra (1979) reported that the methods and techniques that are prevalently applied to estimate the natural groundwater recharge are soil water balance, zero flux plane; one-dimensional soil-water flow model, inverse modeling technique, ground water level fluctuation, hybrid water fluctuation, ground water balance method; isotope and solute profile techniques. Kumar (2000) grouped the methods and the techniques of estimation of groundwater recharge into four groups as empirical methods; groundwater resource estimation; groundwater balance approach and soil moisture data based methods (Seyed et al., 2013; Islam et al., 2014).

The empirical methods are the following equations modified Chaturvedi, Kumar and Seethapathi, UP and Rao formulae. Chaturvedi (1973) derived an empirical equation which expresses recharge as a function of annual precipitation as follows ((Ala-aho et al., 2015; Oke et al., 2017; Natarajan et al., 2018)):

$$R_r = 2.0(P - 15)^{0.4} \quad (1)$$

where: R_r is the net recharge due to precipitation during the year (inches), and P is the annual precipitation(inches).

The Chaturvedi formula was restructured and modified by further work at the U.P. Irrigation Research Institute, Roorkee and the modified Chaturvedi (UP) form of the formula is as follows (Oke et al., 2017):

$$R_r = 1.35(P - 14)^{0.5} \quad (2)$$

Kumar and Seethapathi (2002) established an empirical relationship between groundwater recharge and the corresponding values of rainfall in the monsoon season using the non-linear regression equation of (Oke et al., 2017):

$$R_{rm} = 0.63(P_m - 15.28)^{0.76} \quad (3)$$

where: R_{rm} is the groundwater recharge from rainfall in monsoon season (inch), and P_m is the mean rainfall in monsoon season (inch).

Rao derived an empirical relationship to determine the groundwater recharge in limited climatological homogeneous areas as follows (Ala-aho et al., 2015; Oke et al., 2017; Natarajan et al., 2018):

$$R_r = K(R - X) \quad (4)$$

where R_r is the recharge (mm); K is constant; R is the precipitation (mm), and X is the number of point rainfall. The following boundary empirical equations were applied to different parts of Karnataka as follows:

$R_r = 0.20 (P - 400)$; for regions with annual normal rainfall (P) between 400 and 600 mm;

$R_r = 0.25 (P - 400)$; for zones with P between 600 and 1000 mm

$R_r = 0.35 (P - 600)$; for regions with P above 2000mm, (Ala-aho et al., 2015; Oke et al., 2017; Natarajan et al., 2018).

Islam et al. (2014) gave an empirical relationship between recharge and the corresponding values of rainfall. The relationship is as follows (Oke et al., 2017):

$$R_r = 0.85(P - 27.51)^{0.56} \quad (5)$$

Several other studies conducted on groundwater recharge estimate include Ichwana and Summono (2013), Kung et al. (2013), Rana et al. (2014), Adeleke et al. (2015), Islam et al. (2015), Sabri et al. (2015), and Oke et al. (2015; 2016). Estimating recharge in Nigeria has been a difficult issue due to many factors which comprise the paucity of data, non-availability of mathematical and numerical formulae applicable in the country. Literature (Ala-aho et al., 2015; Oke et al., 2017; Natarajan et al., 2018) stressed the prominence of groundwater recharge in the national development and economics. These discoveries show that there is the need to develop a numerical formula for groundwater estimate in Nigeria. The key objective of this study, therefore was to develop a numerical formula for groundwater estimation that will be applicable in Nigeria, and evaluate its accuracy and applicability using relative error; total error; degree of accuracy, numerical reliability, Model of Selection Criterion (MSC) and Akaike Information Criterion (AIC).

2.0 Materials and Methods

A non-linear regression formula that represents generalization of the simple formulae that relate rainfall to groundwater recharge estimate (depth) was proposed as follows (Oke et al., 2017):

$$R_{gr} = A((P - B))^c \quad (6)$$

where; R_{gr} is the groundwater estimate (depth, mm); A , B and c are the constants for the model and P is the depth of rainfall (mm). The formula was proposed and selected based on the simplicity, nature of the soil, soil cover, infiltration rate, previous studies from literature (Ala-aho et al., 2015; Oke et al., 2017; Natarajan et al., 2018) and a minimum number of fixed parameters to be determined. Monthly rainfall data between 1931 and 2010 for Yola, Nigeria was obtained from literature (Akintola, 1986) and Upper Benue River Basin Development Authority (UBRBDA) meteorological station and used. The rainfall data (annual and monthly) were analysed statistical using Analysis of Variance (ANOVA) and used to calculate the recharge from rainfall using

standard empirical formulae (equations 1 to 5). The constants in the proposed new numerical formula (A, B and c) were determined using Microsoft Excel Solver and groundwater estimated from rainfall. Microsoft Excel Solver was selected based on availability (at no additional cost) and accuracy in numerical solutions. The new formula was used to estimate groundwater recharge from the rainfall in Yola, Nigeria. Relationship between groundwater recharge and year was established.

The accuracy of the developed formula was evaluated using analysis of variance (ANOVA), relative error; a model of selection (MSC) and Akaike Information Criterion (AIC) and compared with selected formulae in use (Rao, Islam et al, 2014, Chaturvedi and modified Chaturvedi formulae). Although, Chaturvedi and the Kumar and Seethapathi formulae were developed using the imperial unit, the groundwater recharge estimated was converted to system international (SI) unit to ensure consistency in the units of measurement (Ala-aho et al., 2015; Oke et al., 2017; Natarajan et al., 2018). Analysis of Variance was conducted using Microsoft Excel package. Procedures employed in the computations of model parameters using Microsoft Excel Solver are as follows (Oke et al., 2017):

Microsoft Excel Solver was added in on the toolbar of Microsoft Excel;

Target (limit, 0, equation (7)) value of the iteration was set for the software based on square of difference as

$$\left[\sum_{t=1}^n R_t - \sum_{t=1}^n A(P_t - B)^c \right]^2 = 0 \quad (7)$$

Changing cells of the iterations were selected, number of iterations, degree of accuracy and maximum time for the iteration were set for the software to meet the target; and

The iteration started through Microsoft Excel Solver as presented in Figure 1.

The Model of Selection Criterion (MSC) is interpreted as the proportion of expected groundwater recharge variation that can be explained by the obtained groundwater recharge. The higher the value of MSC indicates a higher accuracy, validity and the good fitness of the method. MSC can be computed using equation (8) as follows (Oke, 2007):

$$MSC = \ln \frac{\sum_{i=1}^n (Y_{obsi} - \bar{Y}_{obs})^2}{\sum_{i=1}^n (Y_{obsi} - Y_{cali})^2} - \frac{2p}{n} \quad (8)$$

where: Y_{obsi} is the groundwater recharge estimated using UP formula; \bar{Y}_{obs} is the average groundwater recharge estimated using UP formula; p is the total number of fixed constants to be estimated in the equation; n is the total number of groundwater recharge estimated, and Y_{cali} is the groundwater recharge estimated using new formula.

The AIC was derived from the Information Criterion of Akaike (1976). It allows a direct comparison among models with a different number of parameters. The AIC presents the information on a given set of parameter estimates by relating the coefficient of determination to the number of parameters. The AIC and relative errors (RErr) were determined using Eqns 9 and 10 respectively as follows:

$$AIC = n \left(\ln \sum_{i=1}^n (Y_{obsi} - Y_{cali})^2 \right) + 2p \tag{9}$$

$$RErr(\%) = \frac{\sum_{i=1}^n 100 \left(\frac{Y_{obsi} - Y_{cali}}{Y_{obsi}} \right)}{n} \tag{10}$$

3.0 Results and Discussion

Results from this study are discussed in the following categories: rainfall and statistical evaluation, the new formula, estimated groundwater recharges and performance evaluation of the developed formula.

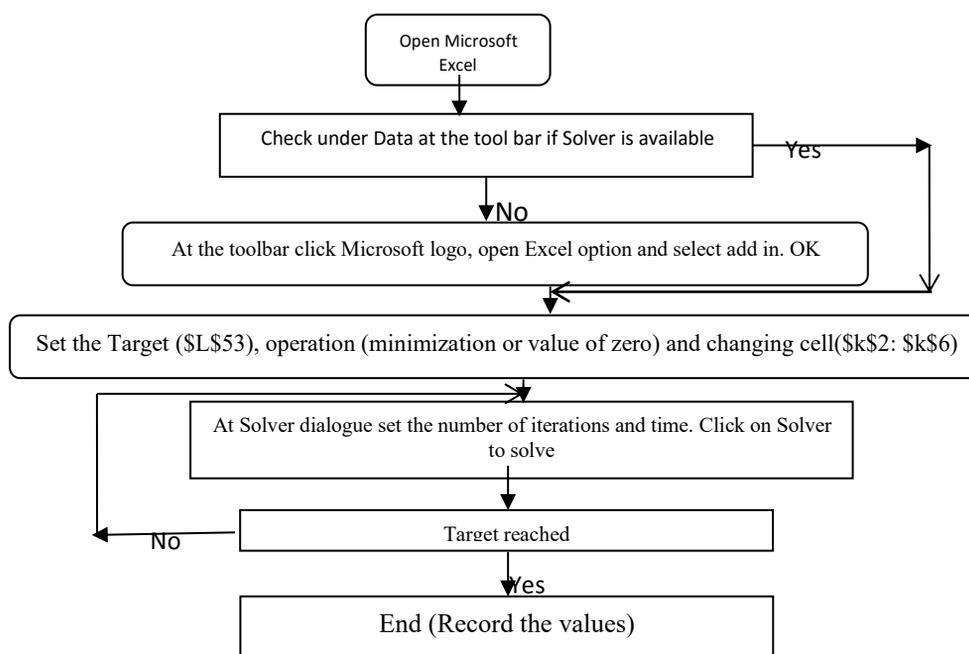


Figure 1: Procedure for using Microsoft Excel Solver in the computation of the

3.1 Rainfall

Figure 2a presents monthly and annual rainfall of the location. From the figure the monthly rainfall varied with the month as well as the year, which indicated that rainfall season in Yola starts from April and ends in the month of October every year. Although there were fluctuations in the amount of rainfall, the range for the month remains valid. Table 1 shows summary of Analysis of Variance (ANOVA) for the monthly and annual rainfalls. The Table revealed that there was a significant difference between the annual rainfall at 95 % confidence level ($F_{76, 836} = 1.099, p = 0.271$). The Table also revealed that there was no significant difference between the pattern of the monthly rainfall for the selected period and location ($F_{11, 836} = 233.562, p = 3.9 \times 10^{-246}$).

3.2 The New Formula

The proposed numerical formula is as presented in equation (11). The values of the parameters were $A = 0.621$; $B = 1.019$ and $c = 0.814$. The value indicates that the new numerical equation for estimating groundwater recharge can be expressed as follows:

$$R_{gr} = 0.621((P - 1.019))^{0.814} \quad (11)$$

This result shows that the new formula is similar to previous numerical formulae in literature such as Oke et al. (2016), Asani et al. (2019). In previous studies, Oke et al. (2017), the numerical formula for groundwater recharge for Abeokuta, Nigeria is as presented in equation (12) with the value of the parameters as $A = 1.673$; $B = 7.219$ and $c = 0.672$.

These constants in the formulae indicated that the numerical equation for estimating groundwater recharge in Abeokuta can be written as follows:

$$R_{gr} = 1.673((P - 7.219))^{0.672} \quad (12)$$

Oke et al. (2016) documented that relationship between rainfall and groundwater recharge in Abeokuta and Ikeja, Nigeria can be expressed as follows:

$$R_{gr} = 0.15P - 102.8 \quad (13)$$

$$R_{gr} = 0.320P - 258 \quad (14)$$

From Oke et al. (2016) correlation coefficients (R_2) for Abeokuta [equation (13)] and Ikeja [equation (14)] are 0.180 and 0.393 respectively, which indicates that these relationships are weak and needs to be improve upon. Lukman et al. (2018) developed relationship between rainfall and groundwater recharge in Sokoto, Nigeria as

$$R_{gr} = 6.814(P - 355.722)^{0.500} \quad (15)$$

These results revealed that no numerical model is the same in values and in magnitude of the constants.

Figure 2b presents relationship between groundwater recharge and period (year). From the figure this relationship is weak, which indicated that there was no strong relationship between year and groundwater recharge.

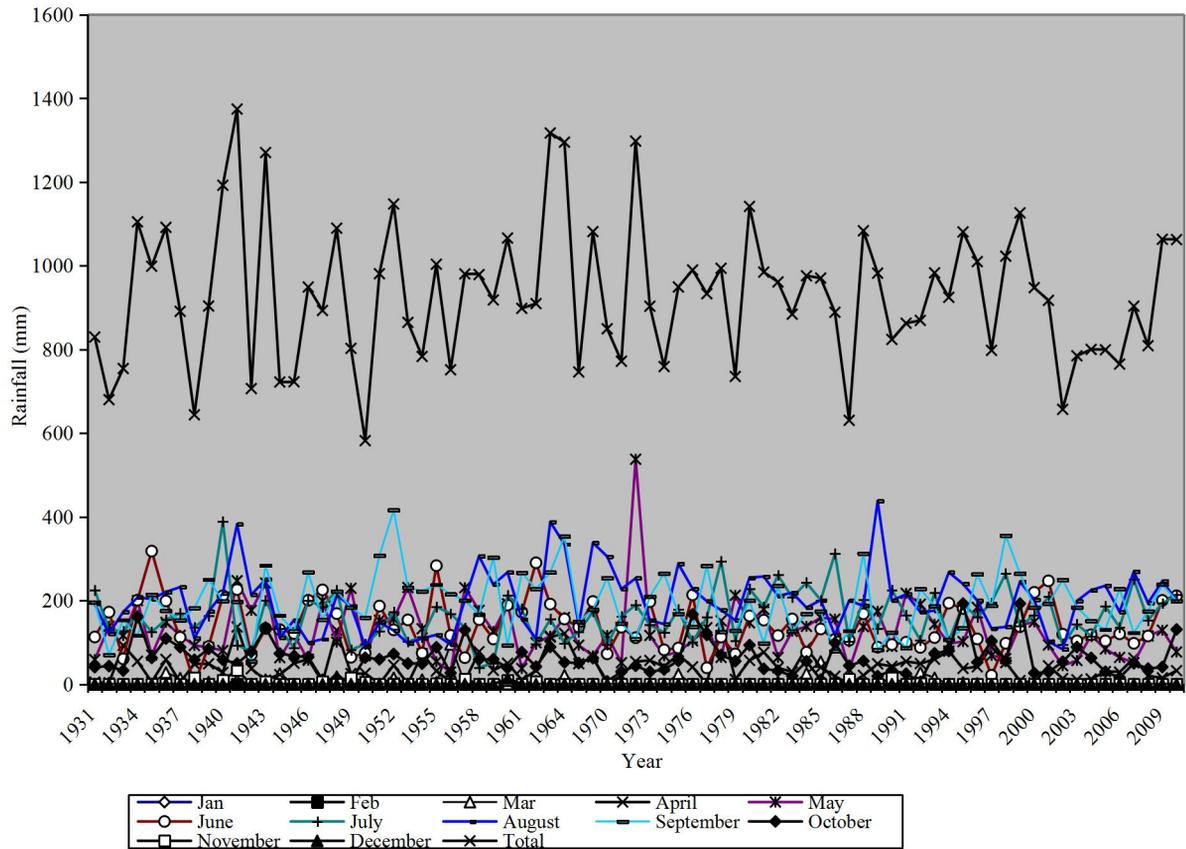


Figure 2a: Rainfall data for Yola, Nigeria over a period of 77 years

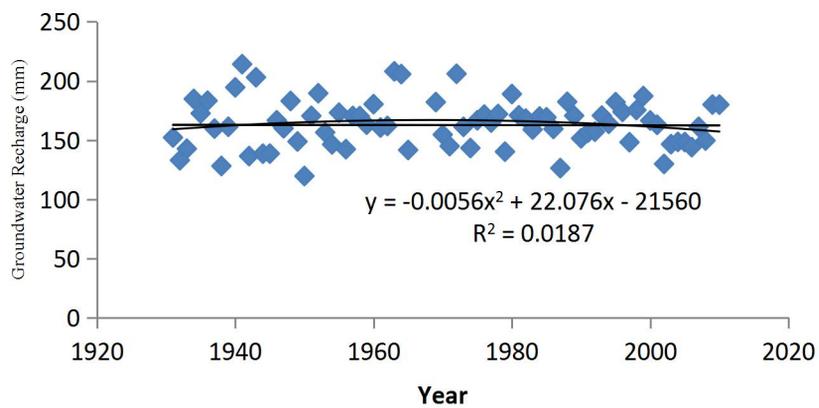


Figure 2b: Relationship between groundwater Recharge and Period

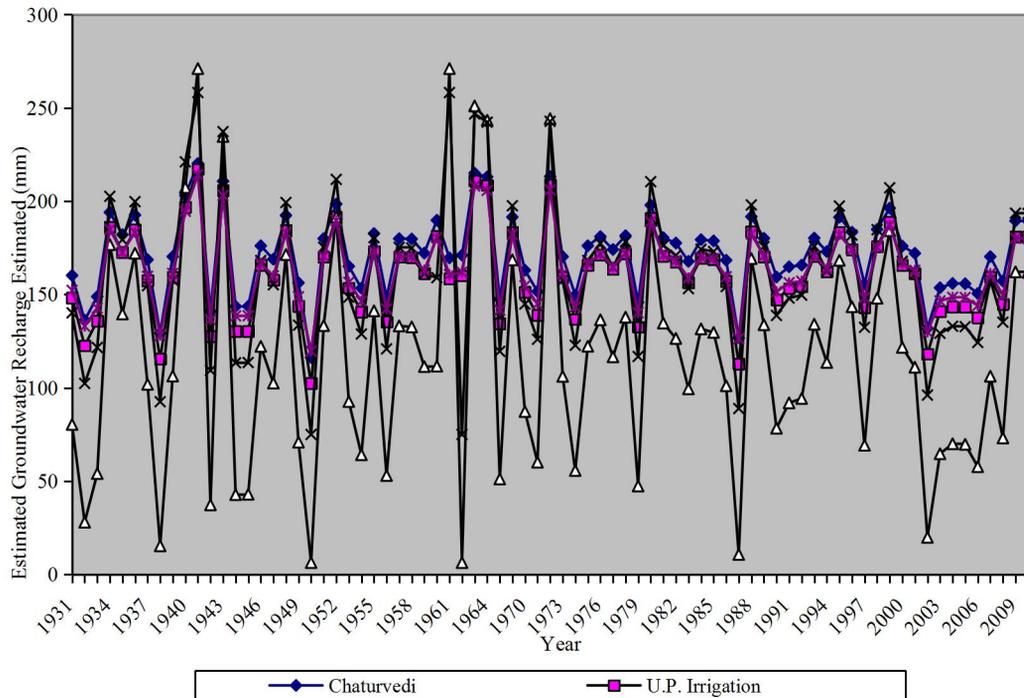


Figure 3: Estimated groundwater Recharge using Various Numerical formulae

Table 1: ANOVA of the Rainfall data in Yola over a period of 77 years

Source of Variation	Sum of Squares	Degree of freedom	Mean Sum of Square	F- Value	P-value
Years	179140.2	76	2357.108	1.098801	0.270962
Months	5511304	11	501027.6	233.5615	3.9×10^{-246}
Error	1793356	836	2145.163		
Total	7483800	923			

3.2.1 Estimated Groundwater Recharges

Figure 3 presents the statistical summary of groundwater recharges estimated using the new numerical formula and formulae that are in use for all the locations. The Figure revealed that groundwater recharge estimated using the Islam et al. (2014) formula had the lowest minimum and the mean in all the cases, while groundwater estimated using the Kumar and Seethapathi's formula had the highest maximum and the mean in all the cases. The implication is that accuracies of these two formulae are lower than other formulae. The Figure also revealed that the lowest standard deviation came from Chaturvedi formula in all the case, while the highest standard deviation came from Rao formula. The result from standard deviation indicates that variation in the groundwater recharge estimated was higher in Rao formula than any other formula. The highest median of computed groundwater recharge came from Rao formula, and the lowest median of computed groundwater recharge came from Islam et al. (2014) formula.

These results and observed in term of median of computed groundwater recharge indicates that Rao formula gave a higher value of groundwater recharge computed than other standard formulae used in the computation of computed groundwater recharge, while Islam et al. (2014) gave the lowest groundwater recharge computed. The implication is that in design and computation of infiltration from rainfall in water budget selection of these two formulae should be done with caution. Results from computed groundwater recharges revealed that there are three empirical formulae exhibit some similarities and closeness in term of computed groundwater recharge. The three empirical formulae are the new formula, modified Chaturvedi and Chaturvedi formulae. The Figure also showed that the estimated groundwater recharges varied with the location as well as the empirical formulae used. This result indicates that accuracy of these formulae is not the same. Table 2 presented result of Analysis of variance of computed groundwater recharge. Analysis of variance of this computed groundwater recharges revealed that there was a significant difference between the estimated groundwater recharges within the formulae and within the year (Table 2) at 99 % confidence level. The F – values range from 14.59 to 106.06 and p- value ranges between 1.49×10^{-66} and 1.29×10^{-54} . This result of ANOVA indicated that is a significant factor in the magnitude of groundwater recharge computation

3.2.2 Performance Evaluation of the New formula

Table 3 presented the performance evaluation of the standard formulae and the new formula. The Table revealed that groundwater recharge computed using Islam et al. (2014) formula provided the lowest MSC and accuracy, as well as the highest AIC and errors (relative error, total and root square errors). From the Table, it was revealed that the highest MSC and Accuracy; lowest errors (total, relative and root square) and AIC were from the new formula. Modified Chaturvedi and Chaturvedi were the next accurate formulae after the new formula, which indicated that the new formula is among accurate formulae. The Table also revealed that there was a slight reduction in the accuracy of these two formulae with reference to the new formula. This slight reduction in accuracy may be attributed to the development of the two formulae (Modified Chaturvedi and Chaturvedi) as an imperial unit's formulae and conversion of the unit from imperial unit to system international unit(SI). The Table revealed that there were three empirical formulae that exhibit some similarities and closeness in respect of computed groundwater recharge. The three empirical formulae are the new formula, modified Chaturvedi and UP formulae. The Table showed the values of MSC were as negative for three standard formulae (Kumar and Seethapathi formula, Rao formula and Islam et al. formula), which indicated that utilization of these three formulae with negative MSC in the groundwater recharge computation should be at the lowest level.

Table 2: ANOVA of the Estimated Groundwater Recharges (Yola)

Source of Variation	Sum of Squares	Degree of freedom	Mean Sum of Square	F- Value	P-value
Years	418215.2	76	5502.832	14.58899	1.49×10^{-66}
Methods	152478.9	4	38119.72	101.0622	1.29×10^{-54}
Error	114666	304	377.1908		
Total	685360.1	384			

Table 3: Results of statistical Evaluations of the models

	Numerical Model	Chaturvedi	Kumar	Rao	Islam et al., (2014)
Relative Error (%)	0.887	6.82	8.80	36.31	59.34
Total Error	267.305	8789.63	34009.90	287532.10	661215.22
Root Square Error	3.47	114.15	441.69	3734.18	8587.21
AIC	436.306	705.26	809.45	973.82	1037.94
MSC	4.911	1.42	0.07	-2.07	-2.90
Accuracy	99.113	93.185	91.198	63.686	40.662

Conclusion

It can be concluded that

The new formula and other two empirical formulae are more accurate than other standard formulae used in this study.

There was a significant difference in the computed groundwater recharges.

Performance evaluation of the formulae revealed that care should be taken in the use of Islam et al, Rao Kumar and Seethapathi formulae in the computation of groundwater recharge based on the value of MSC, AIC and their accuracies.

The new formula is among the best for groundwater recharge estimate based on MSC, AIC and relative error

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