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ORIGINAL RESEARCH ARTICLE

NUMERICAL ANALYSIS OF TWO TERM MODEL USING NEWTON'S METHOD ON DRYING KINETICS OF AN IMPROVED COWPEA (IT 97K-56S-IS)

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ABSTRACT

Invention of computers has made numerical methods prevalent and widely accepted. Different numerical methods exist but all aid in solving large numbers of tedious arithmetic calculations involving complex differential equations and so an analytical solution cannot be applicable, hence, computer simulations and numerical solution become functional. IT 97K-56S-IS, a high yielding, multiple disease resistant cowpea variety with maturity date of 60Days was gotten from the International Institute of Tropical Agriculture (IITA). Experiments were carried out on samples after harvesting at maturity in a convective dryer at drying temperature of 55, 65, 75 and 85ºC and drying kinetics was performed utilising the Two term model. The model was numerically solved using Newton Raphson's iterative method on account that it is a non linear equation, its fast convergence to the roots and requirement of a single guess. A well structured algorithm was written. The performance of the models was evaluated using the coefficient of determination (R^2) and root mean square error (RMSE) showing the relationship between the experimental and predicted moisture ratios. The result shows that Newton Raphson's method would be suitable for Two-Term model if used with a multiplication factor or constant.

1.0 INTRODUCTION

Numerical methods are techniques by which mathematical problems are formulated so that they can be solved with arithmetic operations. Although there are many kinds of numerical methods, they have one common characteristic which is that they invariably have large numbers of tedious arithmetic calculations. Hence, with the development of fast, efficient digital computers, the role of numerical methods in engineering problem solving has increased dramatically in recent years (Steven and Raymond, 2010).

The purpose of modelling is to allow engineers to select the most appropriate method of drying for a given product as well as to determine the most suitable conditions (Aremu *et al.,* 2013). The principle of modelling is based on having a set of mathematical

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equations which can satisfactorily explain the system. Models are often used to study the variables involved in the process, predict drying kinetics of the product and to optimize the operating parameters and conditions (Karathanos and Belessiotis, 1999; Raji and Olanrewaju, 2015). They predict the drying times of several products and also generalize drying curves (Meisami-asl *et al.,* 200; Raji and Olanrewaju, 2015).

A mathematical model can be broadly defined as a formulation or equation that expresses the essential features of a physical system or process in mathematical terms. Differential equations can be used in describing nearly all systems undergoing change (Steven and Raymond, 2010). Thin layer drying process of food products has been described by many mathematical models according to Ojediran and Raji (2010) and these include: cowpea (Jianfang *et al.,* 2013; Raji and Olanrewaju, 2015), green bean and onion (Yaldiz and Ertekin, 2001), millet (Ojediran and Raji), 2010), soybean (Gely and Santalla, 2000), Grains (Tagawa *et al.,* 1996).

High yielding, multiple disease resistant cowpea varieties with varying maturity periods as well as different seed colours, adapted to various Nigerian agro-ecological zones have been developed in IITA, in collaboration with several National Research Institutes and Universities. Several of these varieties have been released in Nigeria and are promoted by the State Agricultural Development Projects (ADPs), farmers' groups and seed companies (Boukar and Ajeigbe, 2010). The most important factor in crop production is the choice of a good variety. Varieties that have resistance to the prevailing biotic and abiotic stresses in the areas should be planted. Maturity, growth pattern, market value, seed size, and seed colour should also be considered in variety selection (DPP, 2011).

The demand for quality farm products increases, therefore, mechanical drying becomes more popular and acceptable as the advantages outweigh the disadvantages. Matured cowpea can thus be harvested promptly and subjected to mechanical drying so as to reduce harvest moisture content to safe level for proper storage (Yakubu *et al.,* 2012). Also, the error present in the solution of models is dependent on how the problem is solved. Numerical methods are used when evaluating empirical information such as experimental data. Such data, no matter how exact and regulated the experiment was conducted, will involve some degree of error. It would be a waste of time to employ exact analytical techniques in such a situation because the answer can never be more "correct" than the input data (Swick, 2013). Therefore, this work investigated the drying kinetics of an improved variety cowpea (IT 97K-56S-IS) at varying drying temperature using the application of numerical method for a complex uncommonly used model (Two term model) thereby developing an efficient algorithm for finding its successive better approximations to the solutions of the complex model.

2.0 MATERIALS

2.1 Sample preparation

A high yielding cowpea, disease resistant, IT 97K-56S-IS with a maturity date of 60days was gotten from the International Institute of Tropical Agriculture (IITA). It was propagated by seeds during the period of a partially wet season on 5th of May, 2014 at the Demonstration Area, Research Farms Unit, IITA, Ibadan, also as stated by Raji and Olanrewaju (2015) and Raji and Olanrewaju (2016). This was to ensure that the seeds tested were harvested at the period needed. For each drying experiment, two hundred (200) grams of the freshly harvested sample was used according to Aremu and Akintola (2014); Tunde-Akintunde and Afon (2009); Raji and Olanrewaju (2015); Raji and Olanrewaju (2016). Each experiment was replicated three times (Aremu *et al.,* 2013) and triplicate samples were spread out in thin layer and placed in the dryer. The drying temperature used for the experiment were 55, 65, 75, 85°C which are within the range of temperatures used by Mario *et al.* (2003), Mc Watters *et al.* (1988) and Wilton *et al.* (2008) for drying of cowpea. The drying process was monitored by weighing the samples every 10mins for the first one hour; then every 30mins for the next three hours and every 1hr for the next three hours till the end of drying according to Ojediran and Raji (2010) Raji and Olanrewaju (2015); Raji and Olanrewaju (2016).

Weighing continued until constant weights were obtained being the period that equilibrium with environment was assumed to have been reached and the test was terminated. Moisture content determined at this point is the dynamic equilibrium moisture content. With the initial moisture already known, weight loss was used to calculate the moisture content using the equation used by Ojediran and Raji (2010) given as:

$$M_t = \frac{M_i m_i - w_i}{m_i - w_i} \quad (1)$$

where, Mt is the moisture content (m.c.) at time t, (% w.b.), M_i ,the initial m.c. (%w.b.), m_i, the initial weight, (g) and w_i is the weight loss at time, t (g). The moisture ratio (MR) of the samples during the thin layer drying experiments was calculated using Equation 2.2 according to Ojediran and Raji (2010):

$$MR = \frac{M - M_e}{M_o - M_e} \quad (2)$$

Matured pods were harvested by hand at period of harvest of 60 Days After Planting (DAP) in line with the recommendation of Directorate Plant Production (DPP, 2011) and to avoid shattering of the pods. Freshly harvested cowpea pods were then cleaned and sorted to remove foreign materials. The initial moisture content of the samples was determined using the drying method. Samples of known weight (200g) were measured with the use of a top loading Scout Pro sensitive weighing scale and were placed in the cabinet tray dryer at 130°C for about 16 – 18 hours as required by ASABE standards (ASABE, 2003) as stated also by Raji and Olanrewaju (2015) and Raji and Olanrewaju (2016).

3.0 METHOD

3.1 Iterative methods

Iterative methods can be used in solving equations by methods of successive approximations to the roots (variable or parameter that satisfy a single nonlinear equation). Three methods of successive approximations include; Newton-Raphson's formula, Bisection method and an algebraic method. Each successive approximation method relies on a reasonably good first estimate of the value of a root being made. One way of determining this is to sketch a graph of the function, say y=f(x), then determine the approximate values of roots from the points where the graph cuts the x-axis. Another way is by using a functional notation method. This method uses the property that the value of the graph of f(x)=0. It changes sign for values of x just before and just after the value of a root (John, 2003).

3.2 Reasons for the choice of Newton Raphson's method

Newton Raphson method was chosen for numerical solution of the Two Term Model due to the following reasons:

1. It can be used for solving nonlinear equations.
2. It converges fast to the roots, if it converges.
3. It requires only one guess.

3.3 Newton's method

The Newton–Raphson's formula, often referred to as Newton's method, may be stated as follows:

If r_1 is the approximate value of a real root of the equation $f(x) = 0$, then a closer approximation to the root r_2 is given by:

$$r_2 = r_1 + \frac{f(r_1)}{f'(r_2)} \quad (3)$$

The advantages of Newton's method over other methods of successive approximations is that it can be used for any type of mathematical equation (i.e. ones containing trigonometric, exponential, logarithmic, hyperbolic and algebraic functions), and it is usually easier to apply than other methods (John, 2003).

3.4 Newton Raphson's Iteration

Let x_0 be a good estimate of r and let r = x_0 + h. Since the true root is r and h = r - x_0, the number, h, measures how far the estimate x_0 is from the truth.

Since h is 'small,' we can use the linear (tangent line) approximation to conclude that

$$0 = f(r) = f(x_o + h)0 \approx \mathrm{f}\,(x_0) + h\mathrm{f}'(\mathrm{x}^0) \quad (4)$$

Therefore, unless f' (x_0) is close to 0,

$$\text{h} \approx -\frac{f(x_0)}{f'(x_0)} \quad (5)$$

It follows that,

$$r = x_0 + h \approx x_0 - \frac{f(x_0)}{f'(x_0)} \quad (6)$$

Then the new improved estimate of r is therefore given by;

$$x_1 = x_0 + \frac{f(x_0)}{f'(x_0)} \quad (7)$$

The next estimate x_2 is obtained from x_1 in exactly the same way as x_1 was obtained from x_0:

$$x_2 = x_1 + \frac{f(x_1)}{f'(x_1)} \quad (8)$$

Continuing in this way. If x_n is the current estimate, then the next estimate x_{n+1} is given by:

n+1is given by;

$$x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)} \quad (9)$$

3.5 Computer Programming and Algorithm

Computer programs are merely a set of instructions that direct the computer to perform a certain task. A well structured algorithm is easier to debug and test, resulting in programs that take a shorter time to develop, test, and update. Structured programming is a set of rules that prescribe good style habits for the programmer. It is flexible enough to allow considerable creativity and personal expression. A key idea behind structured programming is that any numerical algorithm can be composed using the three fundamental control structures: sequence, selection, and repetition (Steven and Raymond, 2010). To keep this description generic, pseudocode was employed.

4 Results and Discussions

4.1 Results

The non linear model of the form (Two Term model):

$$Y = \alpha e^{-k_0 t} + \beta e^{-k_1 t} \quad (10)$$

Solving the non linear regression model using Newton Raphson's Method, the parameters can be estimated using the non linear least square method, the sum of squares is given as:

$$F(x) = \sum (y_i - (\alpha e^{-k_0 t_i} + \beta e^{-k_1 t_i}))^2 \quad (11)$$

The partial derivatives with respect to the parameters `were found and also their second derivatives.

Recall from equation 8, the Newton Raphson formula is given as:

$$x^{n+1} = x^n - \frac{f(x^n)}{f''(x^n)}$$

This can be rewritten as:

$$y^{n+1} = y^n - \frac{g(y^n)}{H(y^n)} \tag{12}$$

Therefore,

$$y^{n+1} = \alpha e^{-k_0 t_n} + \beta e^{-k_1 t_n} - \frac{g(y^n)}{H(y^n)} \tag{13}$$

4.2 Discussions

4.2.1 Drying kinetics of an improved variety cowpea (IT 97K-56S-IS)

The initial moisture content and equilibrium moisture content of (IT 97K-56S-IS) at period of harvest of 60DAP and at drying temperatures of 55, 65, 75 and 85°C is presented in Table 4.1 according to (Raji and Olanrewaju, 2015).

The moisture ratio decreased exponentially with time and the time required to reaching equilibrium moisture content decreases with increasing temperature as illustrated in Figure 4.1. This is a general trend reported for other food products e.g. mulberry, tomatoes, sweet pepper and peach slices. (Doymaz, 2004; Doymaz, 2007: Vengaiah and Pandey, 2007; Kingsly et al., 2007, Raji and Olanrewaju, 2015).

4.2.2 Model Fitting

The model constants and the coefficients for the Two term model at period of harvest of 60DAP and at 55, 65, 75 and 85°C is presented in Table 4.2. The model fittings are illustrated as presented in Figure 4.1a-d.

4.2.3 Limitation of Newton Raphson's Method

Often times, the Newton Raphson's method is very efficient but sometimes there are situations when it performs poorly especially in special case of multiple roots. There is tendency of the Newton Raphson technique to oscillate around a local maximum or minimum. Such oscillations may persist, or as a near-zero slope is reached, whereupon the solution is sent far from the area of interest. Obviously, a zero slope [f(x) = 0] is not suitable because it causes division by zero in the Newton-Raphson formula, it implies that the solution shoots off horizontally and never hits the x axis. Therefore, there is no general convergence criterion for Newton-Raphson. Its convergence depends on the nature of the function and on the accuracy of the initial guess. An initial guess sufficiently close to the root may be required (Steven and Raymond, 2010).

5 Conclusions

To select and apply the most appropriate method for a particular problem, it requires a good understanding of the characteristics of the method and of the problem being solved (Craft, 2010). However, from the results gotten from this research it can be concluded that Newton Raphson's method will be suitable for numerically solving Two-Term model but due to its limitation a multiplying factor (constant) was applied. Other numerical methods may also be applied.

Table 4.1 Initial moisture content and equilibrium moisture content of (IT 97K-56S-IS) at period of harvest of 60DAP and at 55, 65, 75, 85°C

Period of Harvest	Initial M.C. (X100%d.b.)	Equilibrium M.C. (X100%d.b)			
		55°C	65°C	75°C	85°C
60	2.26	0.039	0.028	0.030	0.021

Table 4.2 Drying constants and coefficients of the models for IT 97K-56S-IS at period of harvest of 60DAP

	Multiplication factor	Constants and coefficients	Two term model
55°C	4.1354	a	0.3050
		k	0.6377
		b	-0.0632
		r	0.4128
R^2			0.9423
RMSE			0.0015
65°C	3.3847	a	-0.0716
		k	0.7419
		b	0.3670
		r	0.9754
R^2			0.9053
RMSE			0.0025
75°C	3.5064	a	-0.0684
		k	0.6513
		b	0.3536
		r	0.8861
R^2			0.9839
RMSE			0.0008
85°C	3.4130	a	0.3617
		k	0.7728
		b	-0.0687
		r	0.5356
R^2			0.9946
RMSE			0.0002

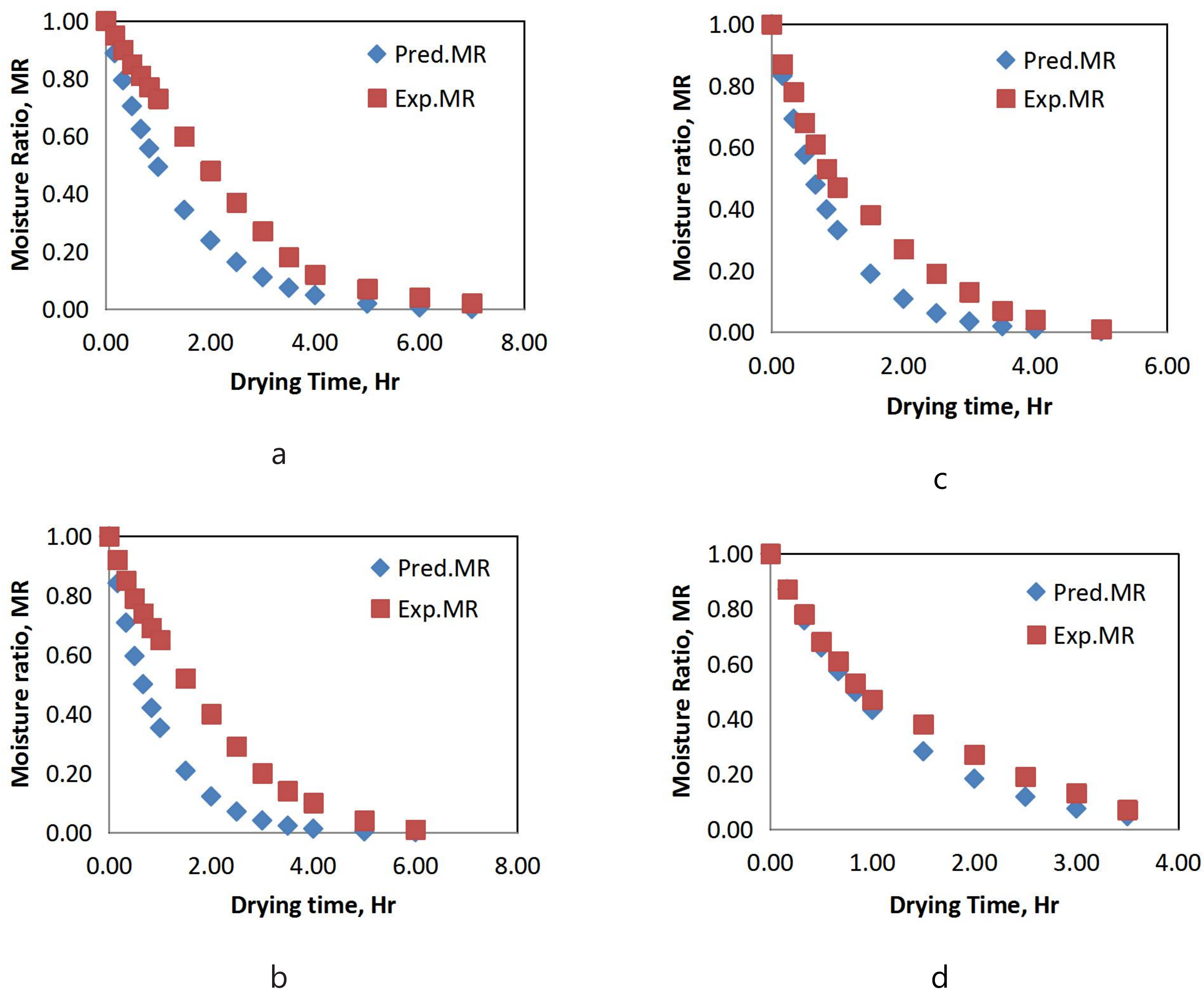

Figure 4.1a: Drying model fittings at period of harvest of 60DAP at 55°C
b: Drying model fittings at period of harvest of 60DAP at 65°C
c: Drying model fittings at period of harvest of 60DAP at 75°C
d: Drying model fittings at period of harvest of 60DAP at 55°C

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