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ORIGINAL RESEARCH ARTICLE

STATE ESTIMATION OF THE NIGERIAN 330KV TRANSMISSION NETWORK USING THE WEIGHTED LEAST SQUARE OPTIMIZATION TECHNIQUE

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ABSTRACT

The state of an electrical power system is the vector of voltage magnitudes and voltage angles at each bus across the entire power system network. The estimates of state variables are very important for online monitoring and control, which are valuable assets in power system operations. The state estimator algorithm is a computational mathematical implementation of a state space technique to process erroneous power system measurements into an estimate of the true power system state vector. It is established, through rigorous research that measurement data obtained from supervisory control and data acquistory systems or Phasor measurement unit are not fit for direct system analysis as they contain errors large enough to give a misrepresentation of the system behavior. To address the issue of erroneous data measurements, this work uses the optimized weighted least square technique to estimate the true state of the power system network. This analysis is achieved by setting up mathematical models of the system network and applying the Weighted Least Square estimation technique to different weights depending on type of measurement. A quantitative and qualitative problem of system observability and error detection in measurement is discussed in this paper. The observability and error quantification process is carried out on the IEEE 14 and the Nigerian transmission grid network through the segmentation of observable islands within the network. This work generates important state results using the MATLAB computational software and run state estimate simulations using the PSAT framework. Using the estimation technique in this work the Nigerian network state space estimation results revealed errors embedded in measurement data with a significant deviation of 1.14 of maximum voltage error in comparison with state estimate result and 16% deviation mean of voltage estimation error in comparison with raw measurement data. The deviation between the raw measurement data and the state estimation results depicts that analysis of power networks without applying that state estimation algorithm may give a misinterpretation of the state variables.

1.0 Introduction

Monitoring and control of the power system is a desirable feature in the operation of power system as it ensures stability and reliability (Kothari, 2010). The monitoring of the generation and transmission has been providing data for dispatch and frequency control however; with the complexity of the power system and the requirement for effective operation, this requirement has become a more difficult task. The Nigerian national grid management and control is carried out manually which makes the system prone to problems; and has resulted in many challenges at generation, transmission and distribution levels (Emodi, 2015; Okonkwo, 1996). The backbone for electrical management is the knowledge of the system state. No control system can effectively tell a power system where to go in future without an adequate knowledge of its present state. Therefore, knowledge of the state of operation of a power system is of maximum importance and can only be achieved by obtaining information from the state vectors and processing these data to estimate the system behavior (Gotti, 2020).

1.1 Data Acquisition in Power Systems

The acquisition of data and data processing for use by the system operator is the fundamental block on which modern power utility control systems are hinged on. A generation of equipment developed to monitor, acquire data, and control functions in the power systems are referred to as the supervisory control and data acquisition (SCADA) systems (Oludele *et al.*, 2016). The SCADA system primary function is to provide a database for the power system which is achieved by real-time base technique.

1.2 Present state of data acquisition system in Nigeria

The transmission company of Nigeria (TCN) attempted to procure and install SCADA systems for the grid however; it has not been very successful. For one, the World Bank financed the procurement of the data acquistory system but could only effectively monitor and retrieve data for just 40% of the entire network (Deloitte, 2021).

It is a fact that system collapse is a regular phenomenon in the Nigerian power system, it is an operational problem. Voltage collapse usually results from the overloading of some injection stations and consequently short circuit faults on transmission and distribution lines. These outages could have been prevented if adequate data were analyzed to predict and control the performance of the system. An analysis of the performance of the Nigerian power system, monitoring, data acquisition and control reveals that the instrumentation level is inadequate for monitoring of the system states for control of the system (Ibe, 2009). For system analysis, the need for sufficient data to predict, investigate disturbances to the possibility of minimizing risk has placed a special responsibility on instrumentation and data acquisition in the power system.

This research paper provides a solution capable of using available data of the transmission state variables to produce an optimal estimate of the static state vector which gives an estimate for unknown parameters lacking measurements. This gives a processed database for monitoring and control decisions in the power system.

2.0 Method

The problem of state estimation is the problem related to the estimate of a random variable **x** from the numerical identity of another related random vector **y** with little statistical information made available for both vectors (Schweppe *et al.,* 1970). The goal of state estimation is to determine the state of the system based on variables been measured, directly or indirectly. In processing errors through state estimation, all measurement errors are assumed to have a statistical property known as a probability distribution with unknown parameters. The joint probability density function for measurements will be derived in relation to the unknown parameters. The joint density function for measurement obtains its peak value labeled by zero when the unknown parameters are chosen closest to their actual values. The Measurement errors are assumed to have a Gaussian (normal) distribution. The parameters for this distribution are its mean, standard deviation and the variance (Pires *et al.,* 1999). The state of a system is the minimal set of variables and the knowledge of these variables in time (t) which together with the input completely determines the behavior of the system. The state vector is the 'n' set of state variables used to describe the nonlinear dynamic equations of a system. Therefore, x is assumed a vector of n random variables (Kothari, 2010)

$$X_1, X_2 \ldots\ldots \ldots X_n \tag{1}$$

And y is another vector of m (< n) random variables

$$Y_1, Y_2 \ldots\ldots\ldots Y_n \tag{2}$$

They are related as $Y = H_X + r$ (3)

Where H is a nonlinear matrix of dimension of m x n that describes the system and r is a zero mean random variable of same dimension of y. The vector 'x' represents the variable to be estimated, while the vector 'y' represents the variable whose numerical values are available

(Pires *et al.,* 2014). This says **y** is linear related to the unknown vector **x** which is corrupted by errors. Assuming state variables to be measured is represented by z, while x represents quantities to be estimated.

$$Z = H_{ij}X_i + H_{ij}X_i + e_i \tag{4}$$

Where, e represents errors between actual measurements z and the true values Z_{true}.
Assuming we have a system with two actual measurement data the matrix of equation (4)

$$Z_1 = h_{11}X_1 + h_{12}X_2 + e_1 = Z_1, \text{true} + e_1 \quad (5)$$

$$Z_2 = h_{21}X_1 + h_{22}X_2 + e_2 = Z_2, \text{true} + e_2 \quad (6)$$

This could also be written as $\left[e = Z - Z_{true} = Z - Hx\right]$ (7)

This says the errors between actual measurement Z and the true values Z_{true} that is approximately H_x of the measured quantity. It is also important to realize that the system variable X cannot be accurate therefore, determination of an estimate can be assumed given by the state $\hat{X}$.

$$\begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} - \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_1 \\ \hat{x}_2 \end{bmatrix} \quad (8)$$

It is also of importance to decide on the criteria required to determine the estimates of the states of $\hat{x}_1$ and $\hat{x}_2$ from which

$$\hat{e} = [\hat{e}_1 \hat{e}_2]^T \quad (9)$$

$$Z = [\hat{z}_1 \hat{z}_2] \quad (10)$$

To ensure that measurements from instruments of known greater accuracy are treated with more emphasis because of their more accurate measurements, each term in the sum of squares is multiplied by an appropriate weighing factor (W) to give the objective function. This brings about the idea of the weighted least square method used for estimation.

As assumed in the equations above considering a system, of four measured quantities the weighted expression will be.

$$F = \sum_{j=1}^{4} W_j e_j^2 = W_1 e_1^2 + W_2 e_2^2 + W_3 e_3^2 + W_4 e_4^2 \quad (11)$$

We select the best estimates of the state variables as those values $\hat{x}_1$ and $\hat{x}_2$ .....$\hat{x}_n$ Which causes the objective function F to take its minimum value this means the estimate of values of X_1 and $X_{2....}$ $X_{n.}$ Which satisfies some set of criteria equations as described in this work. Therefore, for n number of measurement quantities within a power system will be analyzed as,

$$F(x) = \sum_{j-1}^{n} w_j e_j^2 = W_j e_j^2 \quad (12)$$

The nonlinear weighted least algorithm applies the technique of minimizing the square of errors in the measurement data.

$$minimize\ |e|^2 = \sum_{i=1}^{m} \frac{1}{\sigma^2}[z_i - h_i(x)]^2 \quad (13)$$

Where z_i is the ith term of the measurement, e is the error, σ^2 is the variance. H(x) represents the nonlinear relationship with the measurement z_i and the system state. From equation (1)

$$H^T R^{-1}[Z - h(X)] = 0 \quad (14)$$

Where $H_x$the jacobian of h (x) and R is is the covariance matrix, which is the inverse of the system weight. The equation is solved iteratively to obtain estimate of the state vector.

$$[H_X^T (X^K) R^{-1} H_X(X^K)][X^{K+1} - X^K] \quad (15)$$

$$= H_X^T (X^K) R^{-1}[Z - h(x^k)] \quad (16)$$

1. The solution of the iterative process is the set of states x^{k+1} that minimize function $F(X)$.

2.1 Structure of the measurement H matrix

The estimator Jacobian matrix H is not a square matrix it has a (2N -1) where network equals N. while the number of roles in H equals the number of measurement. An assumption made by this work as it relates to the formation of this matrix is that in practical power system network, under steady- state analysis it is assumed that real power flow is less tactful to voltage magnitudes and are much more sensitive to voltage phase angles, while the reactive power flow counterpart is much less tactful to voltage phase angles and are very sensitive to voltage magnitudes. This is done to bring a bit of simplicity to the estimation. The H matrix becomes,

$$H = \begin{bmatrix} \frac{d|v_1|}{d\partial_2} & \frac{d|v_1|}{d\partial_3} & \frac{d|v_1|}{d|v_1|} & \frac{d|v_1|}{d|v_2|} & \cdots & \frac{d|v_1|}{d|v_n|} \\ \frac{d|v_2|}{d\partial_2} & \frac{d|v_2|}{d\partial_3} & \frac{d|v_2|}{d|v_1|} & \frac{d|v_2|}{d|v_2|} & \cdots & \frac{d|v_2|}{d|v_n|} \\ \frac{dpij}{d\partial_2} & \frac{dpij}{d\partial_3} & \frac{dpij}{d|v_1|} & \frac{dpij}{d|v_2|} & \cdots & \frac{dpij}{d|v_n|} \\ \frac{dpij}{d\partial_i} & \frac{dpij}{d\partial_j} & \frac{dpij}{d|v_i|} & \frac{dpij}{d|v_j|} & \cdots & \frac{dpij}{d|v_{jn}|} \\ \frac{dqij}{d\partial_2} & \frac{dqij}{d\partial_3} & \frac{dqij}{d|v_1|} & \frac{dqij}{d|v_2|} & \cdots & \frac{dqij}{d|v_n|} \\ \frac{dqij}{d\partial_i} & \frac{dqij}{d\partial_j} & \frac{dqij}{d|v_i|} & \frac{dqij}{d|v_j|} & \cdots & \frac{dqij}{d|v_{jn}|} \\ \frac{dP_1}{d\partial_2} & \frac{dP_1}{d\partial_3} & \frac{dP_1}{d|v_1|} & \frac{dP_1}{d|v_2|} & \cdots & \frac{dP_1}{d|v_n|} \\ \frac{dP_2}{d\partial_2} & \frac{dP_2}{d\partial_3} & \frac{dP_2}{d|v_1|} & \frac{dP_2}{d|v_2|} & \cdots & \frac{dP_2}{d|v_n|} \\ \frac{dQ_1}{d\partial_2} & \frac{dQ_1}{d\partial_3} & \frac{dQ_1}{d|v_1|} & \frac{dQ_1}{d|v_2|} & \cdots & \frac{dQ_1}{d|v_n|} \\ \frac{dQ_2}{d\partial_2} & \frac{dQ_2}{d\partial_3} & \frac{dQ_2}{d|v_1|} & \frac{dQ_2}{d|v_2|} & \cdots & \frac{dQ_2}{d|v_n|} \end{bmatrix} \quad (17)$$

2.2 State vector Observability criterion.

Observability criterion objective in state estimation is to determine the network Observability, identify the observable island if network is unobservable (Fetzer, 1975). Also, identify branches within the bus where pseudo measurements are necessary to make the system observable. Since the state estimation employs more measurements than the minimum number necessary to define the system, the Observability criterion is employed to determine measurements that improve Observability. Since the measurement Z in a power system are related to the state $\hat{x}$ in a nonlinear way and linearized as in equation 16. A system is said to be completely observable, if and only if the knowledge of the state variable in some finite time after t_o, the knowledge of the state variable description of the system, can be determined by observing the measurement vector Z. A refers [17]

$$Q = [H^T A^T H^T, (A^T)^2 H^T \ldots\ldots\ldots (A^T)^{N-1} H^T] \quad (18)$$

The system is said to be completely observable if the rank of H is equal to the number of variables.

2.3 Error indices

In order to quantify errors within the estimation framework, the maximum error index is applied. Maximum error estimation is the margin of error; it is a measure of the closeness of the estimate to the true value of the state parameter. This bound on error of estimate is usually at a chosen confidence level or interval. The Figure 1 shows the step by step process of the estimator.

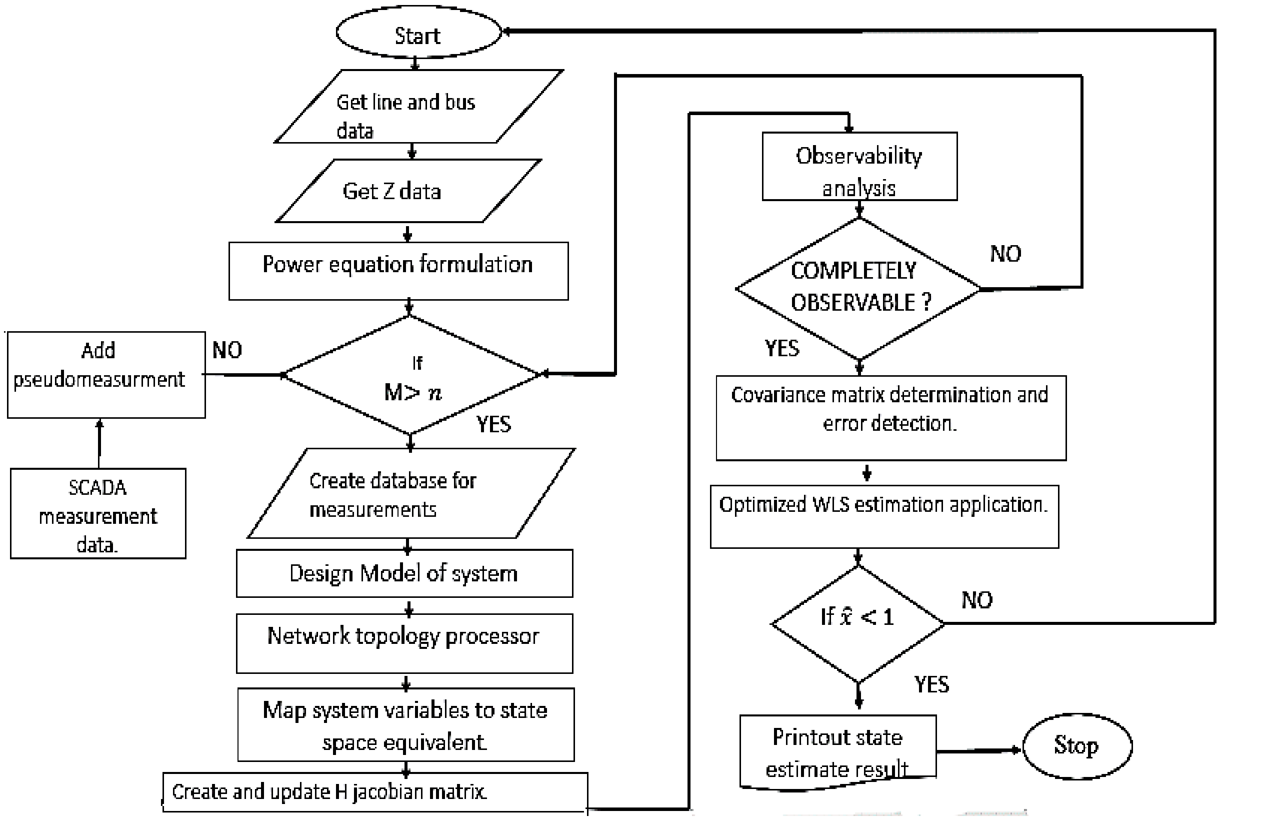

Figure 1: Flow chart of the Weighted Least Square optimization technique

3. Results and Discussion

3.1 Results

This section consists of simulation results based on the software implementation of the weighted least algorithm on the IEEE test bus 14 and the Nigerian 56 buses network. Table 1 shows the voltage magnitudes and voltage phase values which represents the state estimate results for the IEEE 14 bus network. The data used for the IEEE 14 bus analysis is available in (IEEE 14-Bus System(ICSEG), 2021).

Table 1: State estimate result for IEEE 14 bus network in per unit.

BUS	VOLTAGE MAG	PHASE ANGLE	BUS	VOLTAGE MAG	PHASE ANGLE
1	1.0600	0.0000	8	1.0900	-13.3700
2	1.0450	-4.9800	9	1.0561	-14.9500
3	1.0100	-12.7200	10	1.0511	-15.1000
4	1.0186	-10.3200	11	1.0569	-14.8000
5	1.0203	-8.7800	12	1.0550	-15.0800
6	1.0698	-14.2200	13	1.0502	-15.1600
7	1.0619	-13.3700	14	1.0356	-16.0400

In Figure 2, it is shown that each estimated point matches the original base data of the IEEE 14 bus system. This confirms the accuracy of the algorithm in tracking true values of the considered measurement points for estimation as in this case 35 measurement points.

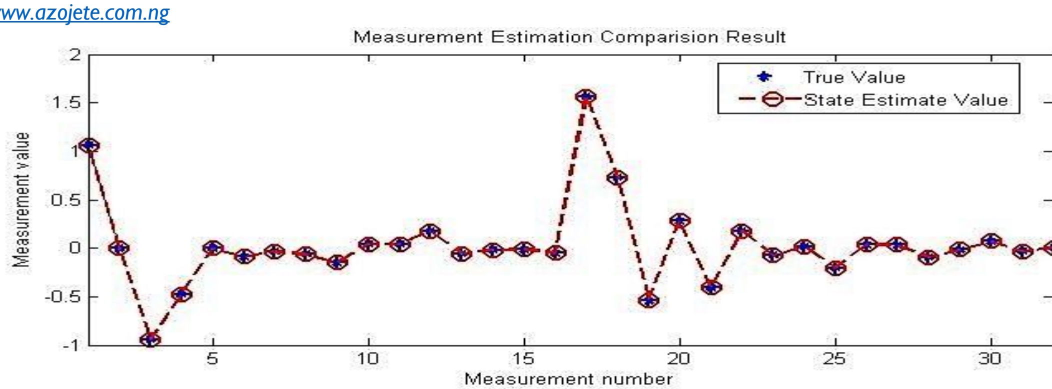

Figure 2: State estimate result comparison against IEEE 14 bus network true data.

While in Figure 3, which shows the voltage angle comparison between the state estimate values and the true value of the voltage phase angle of the IEEE 14 network. It can be seen that the estimator is able to match each of the true values of the network which depicts the accuracy of the estimation process.

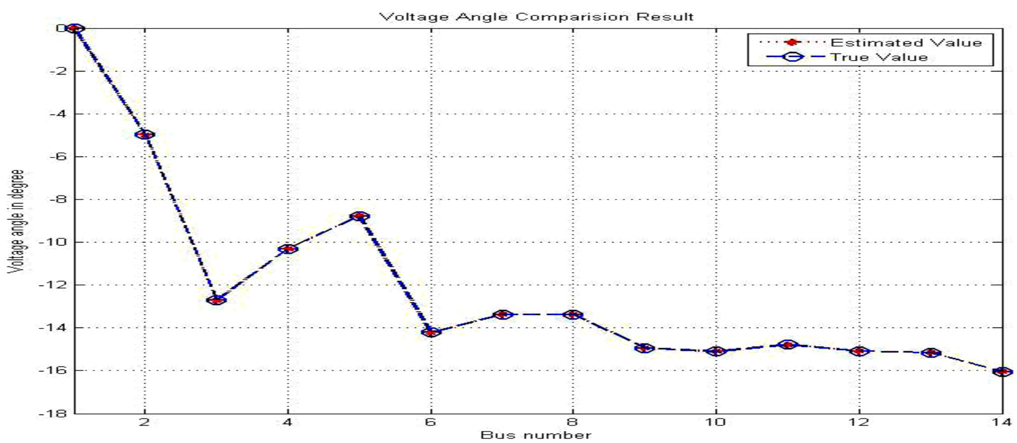

Figure 3: voltage angle comparison estimated value and true value (IEEE14).

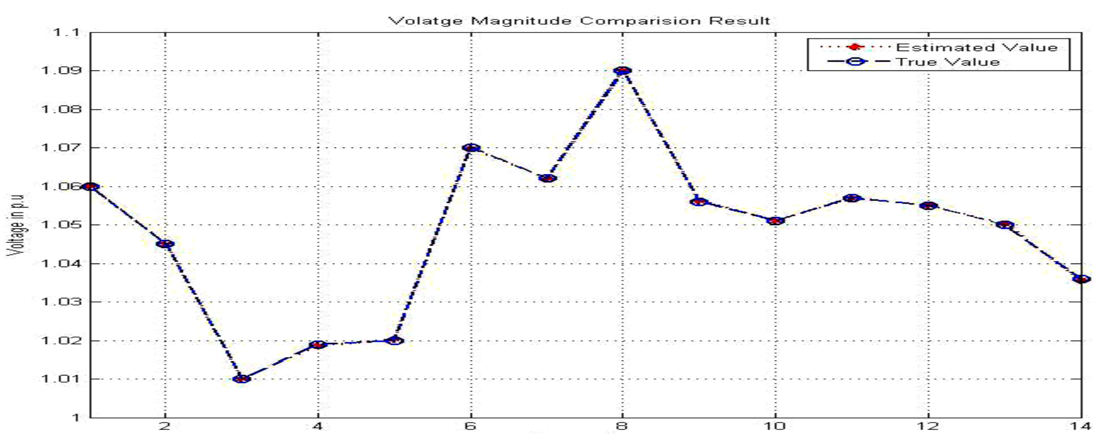

Figure 4: voltage magnititude comparison estimated value and true value.

In the graph in Figure 4 which shows the voltage magnitude state variable comparison, it is seen that estimated points match alongside the true state values of the network. This is however expected as this test system measurement points are considered to be true.

The table 2 shows very small error margin between the true values and the estimated points on IEEE 14 buses which further proves the accuracy of the estimation algorithm to obtain a good estimate of the system state. We obtain values of error as low as 0.0004 and mean error values of 0.0001.

Table 2: Error indices for IEEE 14 buses state estimate.

S\N	Errors	Error index
1	Max voltage magnitudes estimation error(MVMEE)	4.13×10^{-4}
2	Max phase angle estimation error(MPAEE)	6.76×10^{-2}
3	Mean voltage magnitudes estimation error (MVMEE)	1.46×10^{-4}
4	Mean phase angle estimation error(MPAEE)	7.86×10^{-2}

The table 3 shows the state estimation result of the Nigerian 56 bus transmission network.

Table 3: State estimate result for the Nigerian 56 bus network.

Bus number	Voltage magnitude	Phase angle	Bus number	Voltage magnitude	Phase angle
1	0.9127	-0.61	29	1.0837	5.29
2	1.0549	2.48	30	0.8967	-29.01
3	0.912	-1.54	31	1.2862	6.44
4	0.4527	137.33	32	0.9362	-46.8
5	1.0645	5.58	33	1.0178	-14.88
6	1.0274	0.27	34	1.0067	-15.35
7	1.0717	2.62	35	1.0819	1.92
8	1.065	5.06	36	1.1021	-27.78
9	1.0945	1.23	37	1.1438	-25.11
10	0.5144	155.36	38	1.3203	-15.89
11	1.0614	-0.44	39	1.1335	4.55
12	0.9138	-0.44	40	1.2817	-1.71
13	1.0499	-4.00	41	1.0805	4.99
14	0.8926	-10.33	42	1.0444	-3.99
15	1.0989	8.73	43	1.0318	-4.08
16	2.0916	37.64	44	1.1078	-3.81
17	1.0271	-1.15	45	1.1065	-4.18
18	1.1761	7.51	46	1.5544	2.06
19	1.0444	-2.27	47	1.1383	3.17
20	1.0896	5.67	48	1.0659	1.98
21	1.1298	-1.47	49	1.0894	7.22
22	1.1086	-3.78	50	1.0803	4.97
23	1.0811	6.74	51	1.0643	-2.49
24	1.0617	5.54	52	0.7559	-10
25	1.0569	7.56	53	0.8073	-14.16
26	1.0109	3.7	54	1.0947	-1.27
27	0.8103	39	55	1.1136	4.55
28	0.8831	3.29	56	1.1414	0.25

The table 4, shows the error indices for the error margin between the estimated state and the raw measurement values of the network. The MVMEE giving large error margins of 1.1427 and MPAEE error margin of 10.19. For the mean index we obtain a large margin of error between the state estimate and the raw measurement point of 23.52. This indicates that raw measurements carry errors large enough to give a misinterpretation of the system being considered.

Table 4: Error index of the Nigerian 56 buses

S\N	Error	Error index
1	Max voltage magnitude estimation error (MVMEE)	1.1427
2	Max phase angle estimation error (MPAEE)	10.19
3	Mean voltage magnitude estimation error (MVMEE)	0.1607
4	Mean phase angle estimation error (MPAEE)	23.5151

In figure 5, we obtain the comparison of the voltage angle estimated point result and the raw measurement points. It can be seen that there is a large mismatch margin between the estimation results and the raw measurements. This error in measurements arise due to telemetry noise, instrument calibration, digital to analog conversion and human induced factors.

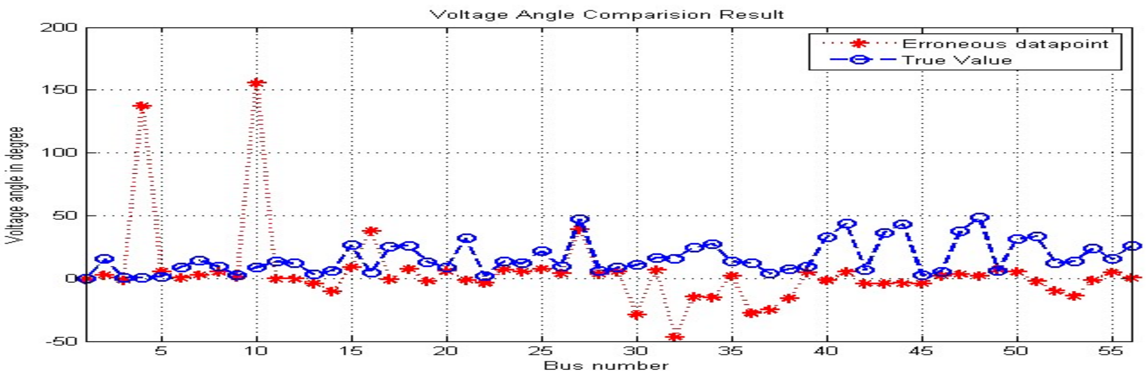

Figure 5: Voltage angle comparison estimated points and erroneous data points.

In figure 6, which shows the voltage magnitude state estimate comparison with the raw measurement we see that at bus 1 we obtain a state estimate voltage magnitude value of 0.9pu whereas the raw measurement says voltage on bus 1 is 0. This shows that without applying the process of state estimation a misinterpretation of the system state will be obtained.

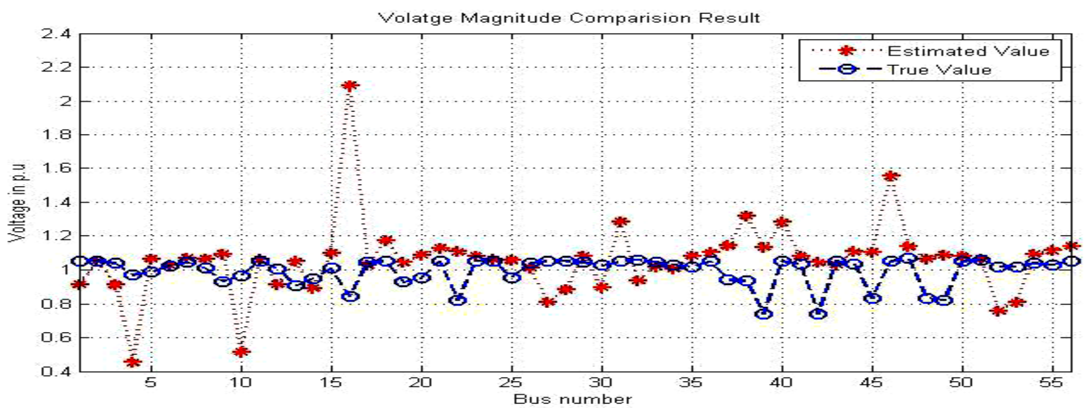

Figure 6: Voltage magnitude comparison estimated values and raw system measurement.

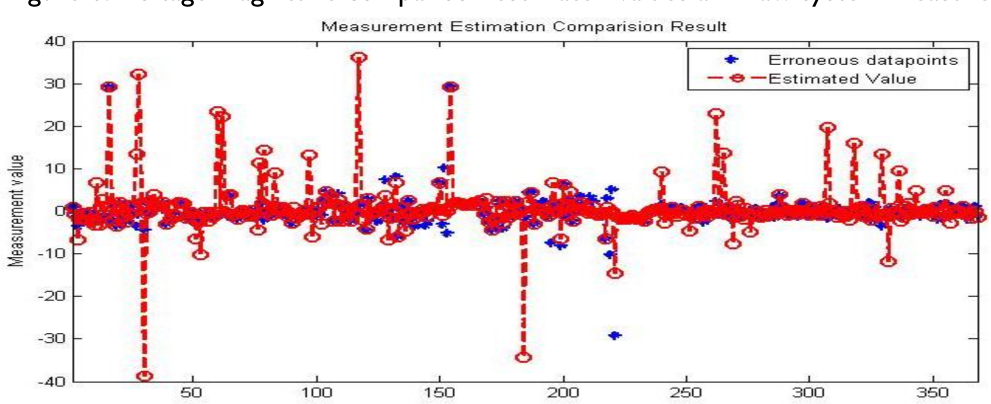

Figure 7: State estimation result comparison with raw erroneous system data for the Nigerian

In the figure 7, the entire measurement data base for the Nigerian system was analyzed in comparison with the state estimate result. For the Nigerian system 400 measurement points where considered for analysis. The graph shows that raw measurement values do not give a true representation of the state of the system.

3.2 Discussion

The estimation process was conducted using the WLS optimized technique shown in Figure 2 and the estimation result was generated as shown in the table 1. Table 1 shows, the estimated voltage and phase angles on each of the buses which signifies the system static state estimate. This indicates the accuracy of the estimation process on this test bus system. Figures 3 and figure 4 show the estimate result of each state (voltage magnititude and phase angle) which confirms the accuracy of the state estimator to estimate the true state of the system.The error indices in table 2 reveals a maximum phase angle estimation error of 6.7613×10^{-4} which further indicates the error reduction effect of the estimator in processing the voltage phase angle. The maximum voltage magnititude estimation error of 4.13×10^{-4} proves that estimation error is at a minimum rate on the state.

In the analysis of the Nigerian 56 bus system, using data obtained from the TCN complete observability could not be achieved. However, in this paper the load flow analysis of the network based on original raw data was carried out the results obtained gave reactive and real power flow on each bus which was then used to further inform the state estimation algorithm.

Pseudomeasurements was used in place of injection power at buses where real and reactive power was zero. This forced the unobservable islands to become completely observable therefore the entire system observable. The additional datapoints made avaliable from the coventional load flow was added to bring in the factor of a bit of integrity into the erroneous raw data. The raw measurements, power flow results and pesudo measurments created a large database of raw measurment data of the Nigerian network. This Database was applied as input to the WLS optimized technique and the state estimate result was obtained as shown in table 3.

Figure 5 shows the comparison of the original erroneous data and the estimated datapoints with respect to the voltage angle. As shown in Figure 6 and Figure 7 there is a large deviation caused by the error in original measurement data. This proves the raw measurement data should not be used directly for power system decisions.

In Table 4, the error index shows a large error in the raw original data in comparison to the estimated datapoint. The max voltage error in comparison with the raw data in table 4 error index shows an error index of 1.14 and max angle estimation of 10.14 . This shows a massive margin of error between the estimated data point and the original raw data point. Table 4 also shows the mean voltage error of 0.1607 and mean angle estimation error of 23.5151. These mean values show another massive error difference embedded in the original data and estimated data points. In obtaining the maximum angle estimation error the algorithm is bounded within a limit so as to ensure the error indices doesn't enter the region of infinity. This mismatch values indicate that the original meaurement data for network do not represent the true state of the system.

4. Conclusion

In this paper, raw measurement data obtained from the transmission company of Nigeria was processed using weighted least square state estimation technique to obtain the true state of the system. The results show that there are errors embedded in conventional SCADA and general data obtained from power system measurement instruments. These errors may arise due to data acquistory process, telemetry noise, instrumentation calibration, weather interference, D/A conversion etc. If power system analysis is carried out without engaging the SE algorithm as proposed in this work, it will yield results that do not reflect the behavior of the system. When decisions within the power system are implemented based on these data, which do not reflect the true system behavior, the power system could be plunged into devastating outcomes.

The complete Observability status for the Nigerian 330KV transmission system was also achieved using available SCADA measurements, employed pseudo-measurements and conventional power flow results of each bus. Using this database, the Nigerian system achieved the status of complete Observability through which the state estimate of its state vectors is realized.

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