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ORIGINAL RESEARCH ARTICLE

OPTIMAL LOAD SCHEDULING OF POWER PLANTS IN A GRID CONSIDERING THE PLANT'S CAPACITY AND LINE LOSSES

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ABSTRACT

In a grid or part of a grid operated by a single service provider with various types of power plants, the total load demand on the grid can be serviced by the plants' outputs in such a way that a minimum cost of energy is spent to generate this total demanded load thereby maximizing profit for the service providers without placing exorbitant tariff on customers. This paper considered how this can be archived in the Niger-Delta region of Nigeria (as a case study). We assumed that the power generated by the plants in the region is also consumed by the region. Using the Lagrnge multiplier optimization method in our analysis and for a total power demand/generation of $2170\ \mathrm{MW}$, a total of $752{,}435 \times 10^4 \mathrm{cal/hr}$ of fuel will be burnt. But with the system of running cost minimization developed in this paper, only about $575{,}521 \times 10^4 \mathrm{cal/hr}$ of fuel was burnt in generating that same quantity of power demanded resulting in a net savings of $176{,}914 \times 10^4\ \mathrm{cal/hr}$. This is equivalent to $2.057 \times 10^3\ \mathrm{kWh}$; with energy sold at ₦48/kWh in Nigeria, a total of ninety-eight thousand, seven hundred and thirty-six Naira (₦98,736.00) only will be saved each hour resulting in a net savings of eight hundred and sixty-four million, nine hundred and twenty-seven thousand, three hundred and sixty naira (₦864,927,360.00) only per annum. This is, therefore, recommended for implementation in running Nigeria's power system.

1.0 Introduction

Short-term gas – hydro – thermal power plants coordination consists of determining the optimal usage of available gas, hydro and thermal resources during a scheduling period of time ranging from one (1) day to one (1) week (Wood *et al*., 2013; Hossain and Shiblee, 2017). This is to determine optimally, which of the generating units should run at any point in time as well as the power generated by the gas, hydro and thermal plants so that the total cost is minimized. Minimizing the total cost in this optimization problem is subject of many control and operational constraints and can be obtained using different optimization methods that include Lagrangian relaxation and Benders decomposition-based methods, Mixed-integer and Dynamic programming among others (Farhat and El-Hawary, 2009; Kovalev *et al*., 2011).

These are however conventional optimization methods which use gradients for the search of optimum values. Evolutionary optimization methods have become an alternative to conventional optimization techniques for solving real world problems having non-convexity, non-differentiability and discontinuity. In this way, evolutionary methods have been successfully applied to power system problems as well. Though, the evolutionary methods have been suitable for power system problems, the premature convergence and stagnation they exhibit poise a problem when using them directly (Attaviriyanupap, *et al*., 2002). A hybrid algorithm combines the conventional and evolutionary optimization techniques for solving power system problems; it can be used for solving the problems having non-convexity and non-smoothness in the function space. This hybrid optimization technique solves problems with multi objectives (Jafari *et al*., 2019; Lei *et al*., 2020; Panda *et al*., 2020).

A cost-based approach has also been used to determine the optimal operational strategy that yields a minimum operating cost. The optimal operational strategy is achieved through the estimation of the hourly generated power, the amount of thermal power recovered from the one type of power plant in the grid e.g fuel cell power plants (FCPP) to satisfy the thermal load, the amount of power trade with the local grid, and the amount of by-products (hydrogen) that can be generated from the plant type which is useful to the society (El-Sharkh *et al*., 2006).

As such, the cost-based optimization is archived by minimizing the objective function (OF) as given in Equation (1)

$$OF = Min\ (\sum_i Cost_i - \sum_i Income_i) \tag{1}$$

Where $\sum_i Cost_i$ is the sum all the cost i, of production (in ₦ or any other currency), $\sum_i income_i$ sum of all i source of income (in ₦ or any other currency).

However, in grids (like Nigeria's) where none of the plant's by-product can be a source of income, such method cannot be adopted.

Power systems operate as a grid (Obi *et al*., 2017) with several power generating plants that include hydropower plants, steam and gas turbine power plants (among others). Suppose the generation and transmission stages of such grid is ran by a single service provider, then the service provider can maximize profit by reducing the cost of production which include generating power and getting the power generated to load centers to the barest minimum (Obi and Offor, 2012). Reduction of production cost can come from the following (Avalos, 2008):

1. Reduction of salaries of technical and non-technical staffs.
2. Reduction of losses along the power lines (before the generated power gets to the consumers so that there will be more power at the load centers).
3. Economic combination of various power plants on the grid to feed the total grid demand (based on their respective running cost).

In this research, the item 3 option given above shall be investigated and we shall assume that the power plants feed only loads in the Nigeria regions under study which are the south – south and south – east regions of Nigeria. Also, we shall assume that the total load demand is less than the total power the various plant combinations can generate. At the moment, there are nine (9) functional power plants in the region and their functional generating capacities are given in Table 1.

Table 1: List of power plants in the region of Nigeria under analysis

Power station	Location	Type	Capacity	Status
Aba Power Station	Aba	Combined cycle gas turbine (more of steam)	340 MW	Operational (260MW)
Afam I – V Power Station	Afam	Combined cycle gas turbine (more of steam)	726 MW	Partially Operational (313MW)
Afam VI Power Station	Afam	Combined cycle gas turbine (more of gas)	624 MW	Operational (600MW)
Alaoji Power Station(NIPP)	Aba	Combined cycle gas turbine (more of steam)	1074 MW	Partially operational (250MW)
Okpai Power Station	Okpai	Combined cycle gas turbine (more of gas)	480 MW	Operational (470MW)
Omoku Power Station	Omoku	Combined cycle gas turbine (more of steam)	450 MW	Operational (340MW)
Sapele Power Station	Sapele	Gas-fired steam turbine and Simple cycle gas turbine (more of steam)	1020 MW	Partially Operational (335 MW)
Sapele Power Station(NIPP)	Sapele	Combined cycle gas turbine (more of gas)	450 MW	Operational (420MW)
Delta - Ughelli Power Station	Ughelli	Combined cycle gas turbine (more of gas)	900 MW	Partially Operational (360 MW)

Source: (Nigerian Agip Oil Company, 2003; Nigerian Agip Oil Company and Rivers state Government, 2003; Shell Petroleum Development Company, 2003; Geometric Power Limited, 2005; Shell Petroleum Development Company, 2005; Federal Government of Nigeria National Integrated Power Project, 2012; Transnational Corporation of Nigeria Plc, 2012; Niger Delta Power Holding Company and Federal Government of Nigeria, 2013; Eurafric Power Limited, 2014; Oputa, 2015).

They are all combined circle power plants generating power from gas and steam respectively. The heat from the gas turbine section is used to heat up water to steam and eventually used to drive another turbine. The more of steam combined circle plant generate more power with the steam driven alternator than the gas. The combined operation of the power plants in Table 1 was carried out without considering the various plant's limit or capacity (Oputa, 2015; Oputa *et al*., 2019). The authors optimized the power plant's operations by using only the respective plant's incremental fuel cost without considering their limit and power line losses.

2. Materials and Methods

In this paper, we developed mathematical models showing the relationship between the power generated by the various power plants in the grid and the cost of fuel burnt to generate such power. We shall then use programs in MATLAB to solve the models developed as they are complex models that will be difficult to solve manually.

The method of equal incremental fuel cost for all power plants in the region under analysis was employed. Hence, we assumed that all 9 plants run with equal incremental fuel cost. We however did not consider the effect of the by-product of each plant to the environment.

If a quantity of fuel burns for one hour liberating an energy f_i (in calorie) to generate a particular power output, P_{Gi} (in MW) in that one hour, the relationship between the quantity of fuel burnt and power generated is given as (Gupta, 2009; Bommirani and Thenmalar, 2013).

$$f_i = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \tag{2}$$

$a_i\ (Cal), b_i\ (Cal/Mw), and\ c_i\ (Cal/MW^2)$ are constants of the ith power plant.

For n plants in the system (nine (9) in this case), the total fuel burnt by all the (9) plants is

$$F_T = \sum_{i=1}^{9} (a_i + b_i P_{Gi} + c_i P_{Gi}^2) \tag{3}$$

Reducing the cost of generating the total power generated by all the plants can be archived by minimizing Equation (3); this equation is thus the operating function (OF).

The minimization of the OF is subject to some factors (constraints). These include:

1. Real power balance: Real power in supply, P_{Gi} must be equal to real power in demand, P_D (plus real power losses, P_L).

$$-\sum_{i=1}^{9} P_{Gi} + P_D + \sum P_L = 0 \tag{4}$$

2. Spare capacity constraint: Some load predictions at load centers are inaccurate; there are also sudden changes in load demand as well as inadvertent losses of schedule generation in the Nigeria power system. The spare capacity constraint can be used to account for these and more. This ensures that the total generation available at any time should be in excess of total anticipated load demand and total system loss by an amount not less than a specific minimum, called the spare capacity, P_{SP} (Bogdan *et al*., 2007; Jinchao *et al*., 2012).

$$\sum_{i=1}^{9} P_{Gi} = \sum P_L + P_{SP} + P_D \tag{5}$$

P_L is calculated in its simplest quadratic for as

$$P_L = \sum_{i=1}^{9} \sum_{j=1}^{9} P_{Gi} B_{ij} P_{Gj} \tag{6}$$

Where, B_{ij} are the loss coefficients connecting the ith and jth buses of the transmission network.

It can also be calculated by a more general formula containing a linear and constant term as shown in Equation (7), this is known as the Kron's loss formula (Saadat, 1999).

$$P_L = \sum_{i=1}^{9} \sum_{j=1}^{9} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{n} B_{0i} P_{Gi} + B_{00} \tag{7}$$

Generation Capacity Constraint: in carrying out the optimization of the OF, the generating capacity of each plant must not be exceeded. Hence,

$$P_{Gi}^{nim} \leq P_{Gi} \leq P_{Gi}^{max} \tag{8}$$

3. Grid Capacity Constraint: For secure operation, the actual transmission capacity must be restricted by its upper limit as

$$S_{ii}(P_{Gi}) \leq S_{li}^{max}. i = 1, b2, \dots, nl \tag{9}$$

Where nl is the number of transmission lines; S_{li} is the electric power flow of the ith transmission line which is influenced by the P_{Gi}; and S_{li}^{max} is the upper limit of the ith transmission line (Gupta, 2009).

4. Minimum Emission of Pollutant from fossil-fuel E_t Constraint: The minimization of the OF should also ensure that the environment is not over polluted with bye-products of the respective plant's operations. Hence, the OF must be minimized in such a way that E_t is minimum (Obi *et al*., 2017).

$$MinE_t = \sum_{i=1}^{Nc} E_i(P_{Gi}) \tag{10}$$

or

$$MinE_t = \sum_{i=1}^{Nc} \left(\alpha_i P_{Gi}^2 + \beta_i P_{Gi} + \gamma_i + \xi_i exp(\tau_i P_{Gi})\right) \tag{11}$$

where α_i, β_i, γ_i, ζ_i, and τ_I, are coefficients of the i[th] generator's emission characteristics, Nc is the number of power plant in the grid.

Our task is therefore to minimize Equation (3) subject to Equations (4) to (10).

However, we shall not consider the effect of emission of pollutant from fossil-fuel and the grid capacity constant. Hence, we shall minimize the OF as shown in Equation (3) subject to constrains as shown in Equations (5), (6) and (8) alone.

Hence, we are to minimize the total cost fuel burnt $F_T = \sum_{i=1}^{9}(a_i + b_i P_{Gi} + c_i P_{Gi}^2)$ subject to

(i) $-\sum_{i=1}^{9} P_{Gi} + P_D + \sum P_L = 0$
(ii) $\sum_{i=1}^{9} P_{Gi} = \sum P_L + P_{SP} + \sum_{i=1}^{9} P_{Di}$
(iii) $P_{Gi}^{nim} \leq P_{Gi} \leq P_{Gi}^{max}$

again, we can also neglect the spare capacity in the constrain in (i) above.

By combining the OF and constrains functions and applying Lagrange multiplier ($\mathcal{L}$) (Saadat, 1999).

$$\mathcal{L} = F_T + \lambda\left((P_D + \sum P_L - \sum_{i=1}^{9} P_{Gi}) + \sum_{i=1}^{9} \mu_{i(\max)}\big(P_{Gi} - P_{Gi(max)}\big)\right) + \sum_{i=1}^{9} \mu_{i(\min)}\big(P_{Gi} - P_{Gi(min)}\big) \tag{12}$$

where $\mu_{i(\max)} = 0$ when $P_{Gi} < P_{Gi(max)}$ and $\mu_{i(\min)} = 0$ when $P_{Gi} > P_{Gi(min)}$.

Thus, if the constraint is not violated, its associate μ variable is zero and the corresponding term in Equation (12) does not exist.

Equation (12) is minimum at

$$\frac{\partial \mathcal{L}}{\partial P_i} = 0 \tag{13}$$

$$\frac{\partial \mathcal{L}}{\partial \lambda} = 0 \tag{14}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{i(max)}} = P_{Gi} - P_{Gi(max)} = 0 \tag{15}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_{i(min)}} = P_{Gi} - P_{Gi(min)} = 0 \tag{16}$$

The solution of the Equation (12) is therefore given as

$$\frac{df_i}{dP_{Gi}} + \lambda \frac{\partial P_L}{\partial P_{Gi}} = \lambda \tag{17}$$

From Equation (7),

$$\frac{\partial P_L}{\partial P_{Gi}} = 2\sum_{j=1}^{9} B_{ij} P_{Gj} + B_{0i} \tag{18}$$

Putting (15) and (2) into (14), (14) can be rewrite as

$$b_i P_{Gi} + 2c_i P_{Gi} + 2\sum_{j=1}^{9} B_{ij} P_{Gj} + B_{0i} = \lambda \tag{19}$$

Or

$$\left(\frac{c_i}{\lambda} + B_{ii}\right) P_{Gi} + \sum_{\substack{j=1 \\ j\neq i}}^{9} B_{ij} P_{Gj} = \frac{1}{2}\left(1 - B_{0i} - \frac{b_i}{\lambda}\right) \tag{20}$$

The optimal dispatch for an estimated λ^1 by solving Equation (20) using Iterative process by gradient method. P_{Gi} and its value at the kth iteration is given as (Saadat, 1999)

$$P_{Gi}^{k} = \frac{\lambda^k(1-B_{0i}) - b_i - 2\lambda^k \sum_{j\neq i} B_{ij} P_{Gj}^{k}}{2\left(c_i + \lambda^k B_{ii}\right)} \tag{21}$$

Total power generated must be equal to the total power demanded and total power lost.

$$\sum_{i=1}^{9} P_i = P_D + P_L \tag{22}$$

Implementing Equation (21) in (22),

$$f\lambda^k = P_D + P_L^k \tag{23}$$

Where

$$f\lambda^k = \sum_{i=1}^{9} P_{Gi}^{k} = \frac{\lambda^k(1-B_{0i}) - b_i - 2\lambda^k \sum_{j\neq i} B_{ij} P_{Gj}^{k}}{2\left(c_i + \lambda^k B_{ii}\right)} \tag{24}$$

Using Taylor series on the RHS of (21)

$$f\lambda^k + \left(\frac{df\lambda}{d\lambda}\right)^k \Delta\lambda^k = P_D + P_L^k \tag{25}$$

$$\Delta\lambda^k = \frac{\Delta P^k}{\left(\frac{df\lambda}{d\lambda}\right)^k} = \frac{\Delta P^k}{\sum\left(\frac{\partial P_{Gi}}{\partial\lambda}\right)^k} \tag{26}$$

where

$$\sum_{i=1}^{9}\left(\frac{\partial P_{Gi}}{\partial\lambda}\right)^k = \sum_{i=1}^{9} \frac{c_i(1-B_{0i}) + B_{ii} b_i - 2\lambda^k \sum_{j\neq i} B_{ij} P_{Gj}^{k}}{2\left(c_i + \lambda^k B_{ii}\right)^2} \tag{27}$$

Note that for any iteration,

$$\Delta P^k = P_D + P_L^k - \sum_{i=1}^{9} {P_{Gi}}^k \tag{28}$$

The iterative process is continued until ΔP^k is less than a specific accuracy limit.

Using the simplest/approximate loss formula

$$P_L = \sum_{i=1}^{9} B_{ii} P_{Gi}^2 \tag{29}$$

Using Equation (29), the common incremental fuel cost of all power plants λ as $B_{ij} = B_{00} = 0$. Then Equations (21) and (27) respectively becomes

$$P_{Gi}^{k} = \frac{\lambda^k - b_i}{2\left(c_i + \lambda^k B_{ii}\right)} \tag{30}$$

$$\sum_{i=1}^{9}\left(\frac{\partial P_{Gi}}{\partial\lambda}\right)^k = \sum_{i=1}^{9} \frac{c_i + B_{ii} b_i}{2\left(c_i + \lambda^k B_{ii}\right)^2} \tag{31}$$

Equations (28) to (31) were used to analyze the system.

3. Results and Discussion

Data collected from respective power plants' data books show the energy or fuel burnt in generating a corresponding amount of power (Oputa, 2015) and they are given in Table 2.

Table 2: Fuel burnt/Power generation Characteristics of each plant

Power Station	Energy (Fuel) Burnt/Hour $[(\text{Cal}) \times 10^4/hr]$	Plant Output Power (Mw)	Power Station	Energy (Fuel) Burnt/Hour $[(\text{Cal}) \times 10^4/hr]$	Plant Output Power (Mw)
Aba Power Station	2.3	0	Delta – Ughelli Power Station	5.1	0
	9,200	80		102,000	300
	11,430	90		105,385	305
	13,900	100		115,845	320
	16,600	110		134,385	345
Afam I – V Power Station	3.1	0	Sapele Power (Station NIPP)	3.2	0
	22,500	150		147,200	400
	28,560	170		150,865	405
	35,345	190		158,330	415
	42,840	210		161,360	419
Afam VI Power Station	2.8	0	Sapele Power Station	4.0	0
	254,000	500		8,725	80
	285,145	530		13,505	100
	295,925	540		15,140	106
	329,460	570		19,330	120
Alaoji Power Station (NIPP)	4.2	0	Omoku Power Station	2.1	0
	36,725	180		8,885	80
	40,857	190		14,810	105
	45,205	200		18,200	117
	47,462	205		21,580	128
Okpai Power Station	3.3	0			
	163,200	400			
	175,548	415			
	195,350	438			
	219,950	465			

Source: (Nigerian Agip Oil Company, 2003; Nigerian Agip Oil Company and Rivers state Government, 2003; Shell Petroleum Development Company, 2003; Geometric Power Limited, 2005; Shell Petroleum Development Company, 2005; Federal Government of Nigeria National Integrated Power Project, 2012; Transnational Corporation of Nigeria Plc, 2012; Niger Delta Power Holding Company and Federal Government of Nigeria, 2013; Eurafric Power Limited, 2014).

The quantity of energy (or quantity of fuel burnt) by each plant when generating power is given in Table 2. From the table, the values of the plant constant- a_i, b_i, c_i for each plant can be calculated and their approximate values are given in Table 3.

The data in Table 2 were used to obtain the fuel cost – power equation any particular plant. For example, using the figures in the blue highlighted values of Aba power station, 92,000kcal/hr generated 80MWin that 1 hour. Hence, $92000 = a + b(80) + c(80)^2$. Also, $139{,}000 = a + b(100) + c(100)^2$

Having some 3 set of equations for a particular plant, 'a', 'b' and 'c' for that plant were obtained and presented in Table 3.

Table 3: Energy equations for various power Plants in the region (a Function of their power Generated)

Power station	a	B	c	Fuel Eqn
Aba Power Station	2.3	19	1.2	$2.3 + 19P_G + 1.2{P_G}^2$
Afam I – V Power Station	3.1	15	0.9	$3.1 + 15P_G + 0.9{P_G}^2$
Afam VI Power Station	2.8	8	1	$2.8 + 8P_G + {P_G}^2$
Alaoji Power Station(NIPP)	4.2	6	1.1	$4.2 + 6P_G + 1.1{P_G}^2$
Okpai Power Station	3.3	8	1	$3.3 + 8P_G + {P_G}^2$
Omoku Power Station	2.1	15	1.2	$2.1 + 15P_G + 1.2{P_G}^2$
Sapele Power Station	4	5	1.3	$4 + 5P_G + 1.3{P_G}^2$
Sapele Power Station(NIPP)	3.2	8	0.9	$3.2 + 8P_G + 0.9{P_G}^2$
Delta - Ughelli Power Station	5.1	10	1.1	$5.1 + 10P_G + 1.1{P_G}^2$

The minimum power to be generated by each individual power plant without economically running at a loss is given in Table 4.

Table 4: Minimum Power to be generated without economic loss

Power station	Minimum Power Generation (MW)	Plant Operational Capacity (MW)
Aba Power Station	20	260
Afam I – V Power Station	38	313
Afam VI Power Station	58	600
Alaoji Power Station(NIPP)	22	250
Okpai Power Station	55	470
Omoku Power Station	45	340
Sapele Power Station	40	335
Sapele Power Station(NIPP)	55	420
Delta-Ughelli Power Station	38	360

Source: (Nigerian Agip Oil Company, 2003; Nigerian Agip Oil Company and Rivers state Government, 2003; Shell Petroleum Development Company, 2003; Geometric Power Limited, 2005; Shell Petroleum Development Company, 2005; Federal Government of Nigeria National Integrated Power Project, 2012; Transnational Corporation of Nigeria Plc, 2012; Niger Delta Power Holding Company and Federal Government of Nigeria, 2013; Eurafric Power Limited, 2014).

The minimum power that a particular power plant must be generating to keep the said plant in the system is got from the fact that most plant in the system are combined cycle power plants. Thus, it comprises of more than one unit and some unit's operation depend on the operations of some other units.

From Tables 3 and 4, the respective incremental fuel cost and the plant's limits are given in Table 5:

Table 5: Incremental Fuel Cost model of various power plant in the region

S/N	Power station	Incremental λ	Plant's Limit
1	Aba Power Station	$[19 + 2.4P_G]$	$20 \le P_G \le 260$
2	Afam I – V Power Station	$[15 + 1.8P_G]$	$38 \le P_G \le 313$
3	Afam VI Power Station	$[8 + 2P_G]$	$58 \le P_G \le 600$
4	Alaoji Power Station(NIPP)	$[6 + 2.2P_G]$	$22 \le P_G \le 250$
5	Okpai Power Station	$[3.3 + 2P_G]L_5$	$55 \le P_G \le 470$
6	Omoku Power Station	$[15 + 2.4P_G]$	$45 \le P_G \le 340$
7	Sapele Power Station	$[5 + 2.6P_G]$	$40 \le P_G \le 335$
8	Sapele Power Station(NIPP)	$[8 + 1.8P_G]L_8$	$55 \le P_G \le 420$
9	Delta-Ughelli Power Station	$[10 + 2.2P_G]$	$38 \le P_G \le 360$

Taking a case where the 9 power plants generate a total power of 2,170 MW at a particular time from Table 2; we calculated the approximate lost coefficient of the system as given in Table 6 and they are specified in per unit on a 1,000 MWA base.

Table 6: Lost coefficient of the system under analysis

Line Coefficient	Value ($\times 10^{-6} MW^{-1}$	Line Coefficient	Value ($\times 10^{-6} MW^{-1}$)
B_{11}	1.46	B_{22}	2.17
B_{33}	1.15	B_{44}	1.73
B_{55}	2.22	B_{66}	2.68
B_{77}	1.99	B_{88}	2.73
B_{99}	1.68		

Then the simplest power loss for the system as given in Equation (29);

$$P_L = 0.0146\left(\frac{P_{G1}}{100}\right)^2 + 0.0217\left(\frac{P_{G2}}{100}\right)^2 + 0.0115\left(\frac{P_{G3}}{100}\right)^2 + 0.0173\left(\frac{P_{G4}}{100}\right)^2 + 0.0222\left(\frac{P_{G5}}{100}\right)^2 + 0.0268\left(\frac{P_{G6}}{100}\right)^2 + 0.0199\left(\frac{P_{G7}}{100}\right)^2 + 0.0273\left(\frac{P_{G8}}{100}\right)^2 + 0.0168\left(\frac{P_{G9}}{100}\right)^2 \quad (32)$$

Starting with λ^1 as 400.0, the values for $P_{G1}^1, P_{G2}^1, \ldots, P_{G9}^1$ from Equation (30), and P_L from Equation (32) were computed by a program developed in MATLAB and the results are as presented in Table 7 in MW (where $P_{G1}^1, P_{G2}^1, \ldots, P_{G9}^1$ are in the order as presented in the Table 5 listing the power plants, i.e P_{G1}^1 power generated by Aba power station, P_{G2}^1 is the power generated from Afam I – V Power Station and so on)

Table 7: Power plant generating power after 1st iteration

Plant generation	Value (MW)	Plant generation	Value (MW)	Plant generation	Value (MW)
P_{G1}^1	158.6728	P_{G2}^1	213.6828	P_{G3}^1	195.9099
P_{G4}^1	178.9783	P_{G5}^1	195.8261	P_{G6}^1	160.2735
P_{G7}^1	151.8301	P_{G8}^1	217.5139	P_{G9}^1	177.1645
P_L^1	0.7215				

Total power 2,170 MW
Real power difference from the 1st iteration is therefore
$\Delta P^1 = 2170 - \sum_{i=1}^{9} P_{Gi}^1 - P_L^1 = 511.6009$

From equation (31),

$\sum_{i=1}^{9} \left(\frac{\partial P_{Gi}}{\partial \lambda}\right)^1 = 4.1157$

$\Delta\lambda^1 = \frac{511.6009}{4.1157} = 124.305$

$\lambda^2 = \lambda^1 + 124.305 = 524.305$

Repeating the process for the second iteration (using $\lambda^2 = 524.305$) gives the results displayed in Table 8

Table 8: Power plant generating power after 2nd iteration

Plant generation	Value (MW)	Plant generation	Value (MW)	Plant generation	Value (MW)
P_{G1}^2	210.4095	P_{G2}^2	283.5900	P_{G3}^2	257.9969
P_{G4}^2	235.3991	P_{G5}^2	258.8524	P_{G6}^2	211.9622
P_{G7}^2	199.6725	P_{G8}^2	287.3723	P_{G9}^2	234.5880
P_L^1					0.8632

Real power difference from the 2nd iteration is therefore
$\Delta P^2 = 2170 - \sum_{i=1}^{9} P_{Gi}^2 - P_L^2 = -4.8632$

Again, from Equation (31),

$\sum_{i=1}^{9} \left(\frac{\partial P_{Gi}}{\partial \lambda}\right)^2 = 4.0081$

$\Delta\lambda^2 = \frac{-4.8632}{4.0081} = -1.2113$

$\lambda^3 = \lambda^2 - 1.2113 = 523.0937$

Repeating the process for the third iteration (using $\lambda^3 = 523.0937$) gives the results displayed in Table 9

Table 9: Power plant generating power after 3rd iteration

Plant generation	Value (MW)	Plant generation	Value (MW)	Plant generation	Value (MW)
P_{G1}^3	209.9055	P_{G2}^3	282.6187	P_{G3}^3	257.3920
P_{G4}^3	234.8494	P_{G5}^3	258.2481	P_{G6}^3	211.4587
P_{G7}^3	199.1074	P_{G8}^3	286.7098	P_{G9}^3	233.6382
P_L^3					0.8461

This iterative process continues until the 6th and 7th iterations.
$\Delta P^6 = 2170 - \sum_{i=1}^{9} P_{Gi}^3 - P_L^6 = -0.4036$
Again, from Equation (31),
$\sum_{i=1}^{9} \left(\frac{\partial P_{Gi}}{\partial \lambda}\right)^6 = 3.7465$
$\Delta\lambda^6 = \frac{-0.4036}{3.7465} = -0.1077$

$\lambda^7 = \lambda^6 - 0.1077 \ = 518.7254$
$as\ \lambda^6 = 518.8331$
The 7th iteration gives results presented on Table 10:

Table 10: Power plant generating power after 7th iteration

Plant generation	Value (MW)	Plant generation	Value (MW)	Plant generation	Value (MW)
P_{G1}^7	209.1435	P_{G2}^7	282.1315	P_{G3}^7	256.124
P_{G4}^7	234.3539	P_{G5}^7	257.8456	P_{G6}^7	211.2426
P_{G7}^7	198.5624	P_{G8}^7	286.1638	P_{G9}^7	233.5374
P_L^7					0.7653

$\Delta P^7 = 2170 - \sum_{i=1}^{9} P_{Gi}^3 - P_L^7 = -0.3815$
We settle for the results on table 9 as our final iterative results since the difference is below 4 kW (and $4\ kW \ll 1270\ MW$).

The total fuel consumption worth of the total power plants to generate the 2170 MW (including losses) at normal generation and for optimum power generation are given in Equations (33) and (34) respectively.

$$F_{Tn} = f_1(80) + f_2(150) + f_3(500) + f_4(180) + f_5(400) + f_6(80) + f_7(80) + f_8(400) + f_9(300) \quad (33)$$

$$F_{Ted} = f_1(209.1435) + f_2(282.1315) + f_3(256.624) + f_4(234.3539) + f_5(257.8456) + f_6(211.2426) + f_7(198.5624) + f_8(286.1638) + f_9(233.5374) \quad (34)$$

The power generated by the various plants and fuel/energy burnt per hour when the total generation is shared as highlighted in Table 2 and when shared with the plants running on equal incremental fuel cost is contained in results of Equations (33) and (34) are presented in Table 11.

Table 11: Comparing sharing of 2,170 MW by the various plants in grid

POWER STATIONS	WITH UNEQUAL INCREMENTAL FUEL COST		WITH EQUAL INCREMENTAL FUEL COST	
	Fuel Burnt per Hour [× 10^4Cal/hr]	Plant Output Power [MW]	Fuel Burnt per Hour [× 10^4Cal/hr]	Plant Output Power [MW]
Aba Power Station	9,200	80	56,413	209.1435
Afam IV-V Power Station	22,500	150	75,299	282.1315
Afam VI Power Station	254,000	500	67,860	256.124
Alaoji Power Station (NIPP)	36,725	180	61,772	234.3539
Okpai Power Station	163,200	400	67,976	257.8456
Omoku Power Station	8,885	80	56,667	211.2426
Sapele Power Station	8,725	80	52,252	198.5624
Sapele Power Station (NIPP)	147,200	400	75,471	286.1638
Delta-Ughelli Power Station	102,000	300	61,811	233.5374
TOTAL	752,435	2170	575,521	2169.6185

With a total power of 2,170 MW generated by all nine (9) generating units in the region to service the power demand and losses along the power lines, the energy burnt by each plant and the total energy burnt by all the 9 power plants in every hour are presented in Table 10. Using a case where the plant's individual generation as highlighted on Table 2 equals 2170 MW, their individual fuel burnt are shown in Table 10 and the total fuel burnt by all nine (9) plants in the generation of this power, is $752{,}435 \times 10^4\ Cal/hr$. However, when the plants ran with the same incremental fuel cost, the power generated by each plant changed. More generating responsibilities were given to those plants whose generation costs are relatively lower. For example, Afam IV-V and Aba power stations (a more of steam combine circle plants) were given more generating responsibilities as they are relatively cheaper generating with them than most other plants on the grid. Hence, their generations were increased from 150 MW to 282.1315 MW and from 80 MW to 209.1435 MW respectively. However, the same total power generation of approximately 2170 MW was achieved (precisely 2169.6185 MW). The energy used by the individual plant and the total energy used by all nine (9) plants to generate that same total of 2,170 MW power each hour as presented in Table 10 is $575{,}521 \times 10^4\ Cal/hr$. This implies that a total of $176{,}914 \times 10^4\ Cal$ (or $7.407 \times 10^9\ Joules$) of energy is saved each hour. This value converted to kWh gives $2.057 \times 10^3\ kWh$ as the energy saved each hour. With energy sold at forty-eight naira only per kilo-watt-hour (₦48/kWh) in Nigeria, a total of ninety-eight thousand, seven hundred and thirty-six naira (₦98,736.00) only will be saved each hour. This means that a total of eight hundred and sixty-four million, nine hundred and twenty-seven thousand, three hundred and sixty naira (₦864,927,360.00) only will be saved per annum.

4. Conclusion

This paper showed how the total running cost of different power generating plants in a grid feeding a fixed load can be achieved. This was done by giving more generation task to those power plants whose cost of generating power is relatively low (provided their generation capacity is not exceeded). With this, the same quantity of total power is generated with a relatively lower cost; this can be regarded as comparative cost advantage in power generation cost. As seen from the studies, a total of over eight hundred million (₦800,000,000:00) naira was saved annually when all power plants in the region analyzed were ran with this method (equal incremental fuel cost) over when the load was shared arbitrarily by the plants. This saved money can be used to build more power plants, expand transmission capacities or used for other capital projects in the country. Therefore, optimized combination of power plant running is highly recommended in running the Nigerian power system.

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